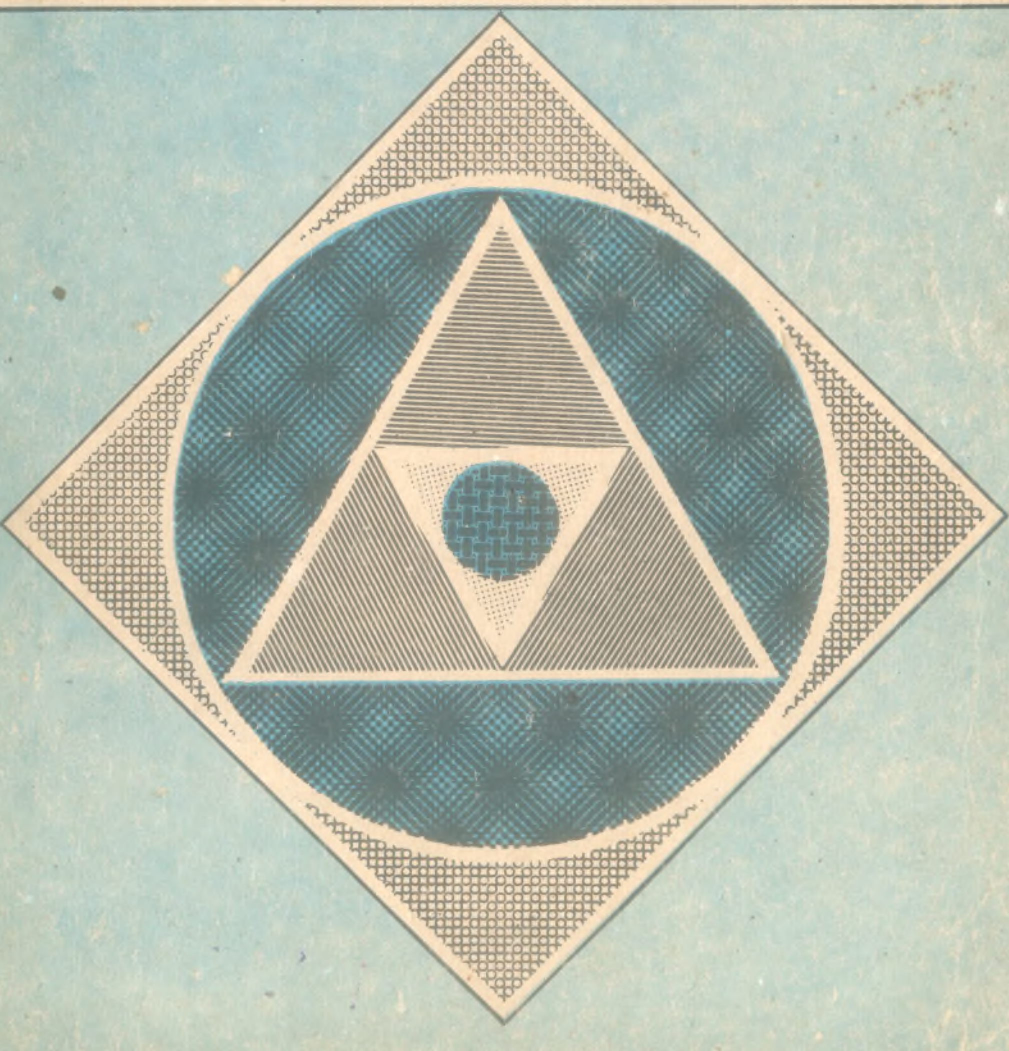
GEOMETRY

STANDARD X



GEOMETRY

STANDARD X

MAHARASHTRA STATE
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PLEDGE

"India is my country. All Indians are my brothers and sisters.

I love my country and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness."

PREFACE

Taking into account the needs, the age groups and the understanding capacity of students belonging to all strata of society, the Maharashtra State Board of Secondary and Higher Secondary Education, having given a second look to the objectives of mathematics courses at all school levels, has reframed the syllabus of Mathematics from Std. VIII to X.

The textbooks of mathematics for Std. VIII and Std. IX, which were prepared as per the revised syllabus, were already published. In continuation of that series, the textbooksof mathematics for Std. X is prepared. It is hoped that the book finds approval from students as well as from teachers.

While writing this book main attempt has been to make students aware of the fact that mathematics is closely related with day-to-day life. Attempt is also made to achieve the following objectives.

- ★ To awaken and encourage the inquisitive mind in students.
- ★ To develop students' ability of logical thinking, so as to enable them to put ideas in precise language and to create liking towards it.
- ★ To make them appreciate the interrelation of ideas which are responsible for growth of Mathematics.
- ★ To make them aware that mathematics is essential for deeper understanding of all sciences.
- ★ To make them realise that mathematics has its own language, which they should be able to understand, to speak, and to write.
- ★ To develop the capacity of students to solve examples and problems.

The matter in this textbooks is so presented as to establish a good conversation between the content matter of Std. IX and Std. X, and to facilitate the revision of the main concepts learned in Std. IX to help the students for better understanding of new concepts to be introduced in this textbooks.

In geometry textbooks, the proofs of a number of theorems, corollaries and their converses contained in the syllabus are given for better understanding of the subject. But, the students are not expected to reproduce proofs of all of them. The proofs of those theorems, which are expected from the students, are specifically marked in the syllabus by a star. While studying the textbooks, students as well as teachers should note this fact.

Prior to publication, the script of this textbook was sent to renowned educationists, to experienced teachers and to wellknown authors of books in mathematics and their criticism, suggestions and opinions were solicited. This textbook is finalized after taking into consideration their suggestions and criticism. The Board is whole heartedly thankful to all these persons for their sincere suggestions and constructive criticism. The Board is grateful to the editor, co-ordinator and members of the Board of Studies appointed by the Board; who have extended their co-operation by putting their hard work with devotion in the preparation of this book.

The Board also thanks the Director of the Maharashtra State Bureau of Textbook Production and Curriculum Research, for his co-operation in bringing out this book. Thanks are also due to the Controller of the Bureau for getting it printed in a nice and attractive form.

Chairman,
Maharashtra State Board of Secondary
& Higher Secondary Education,
& Addl. Director of Education
Maharashtra State, Pune-411 010.

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Chapter 1

THEOREM OF PYTHAGORAS

§ 1.1 It is said that the famous Greek Philosopher Plato had displayed the following notice on the door of his study-room, 'A person totally ignorant of geometry need not venture to enter this room.' This indicates the extent of importance that was given to study of geometry in the ancient Greek civilization.

Another Greek mathematician named Pythagoras lived in the sixth century B.C. There is a theorem in geometry which goes by his name. It is called theorem of Pythagoras.

The geometrical property stated in his theorem is often used in higher mathematics and has wide applications. We are going to study this theorem having such a basic importance.

However, the property stated in this theorem was known to Indians. Shulva (800 - 500 B.C.), Boudhayana (800 B.C.) and Katyayana (500 B.C.) have mentioned this property in their literature.

§ 1.2 Some preliminaries:

(i) if a square is constructed so that its side is a given segment, that square is called the square on the given segment. e.g. Square I in fig 1.1, is a square on the segment AB.

(ii) Some words in geometry have two connotations. The word **'side'** in geometry means a segment as well as the length of the segment. The meaning of the word has to be understood with reference to its context.

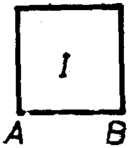


Fig. 1.1

(iii) Just as we denote the vertices of a triangle by capital letters such as A,B,C,... the lengths of the sides opposite to these vertices are denoted by corresponding small letters a,b,c...

For example;
In △ ABC, $l(BC) = a$, $l(AC) = b$ and $l(AB) = c$

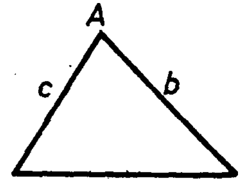


Fig. 1.2

In Δ XYZ, $l(YZ) = x$, $l(XZ) = y$ and $l(XY) = z$.

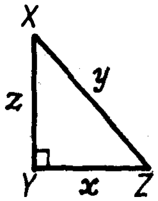


Fig. 1.3

§ **1.3 Theorem of Pythagoras** is about the area of the squares on the sides of a right angled triangle.

The statement of the theorem is as follows:

In a right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

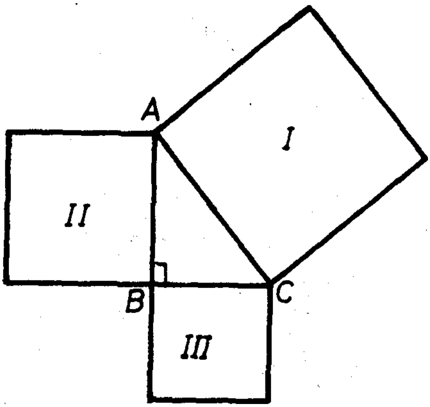


Fig. 1.4

In fig. 1.4, Δ ABC is a right angled triangle. $m\angle B = 90°$. Squares on each side of Δ ABC are marked as I, II and III. By theorem of Pythagoras, A(square I) = A(square II) + A(square III).

See fig. 1.5.

In Δ ABC, $m\angle C = 90°$, using the convention of small letters, c is the length of the hypotenuse. Area of the square on the hypotenuse is c^2. Similarly, the areas of the squares on the other two sides will be a^2 and b^2 respectively.

We now, state the theorem of Pythagoras as follows:

'If in Δ ABC, $m\angle C = 90°$, then $c^2 = a^2 + b^2$.'

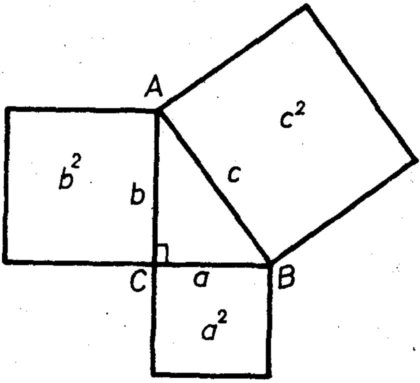


Fig. 1.5

If, however, the theorem is stated in the above form its reference to geometrical concept of area goes in the back ground, since c^2 is read as c squared.

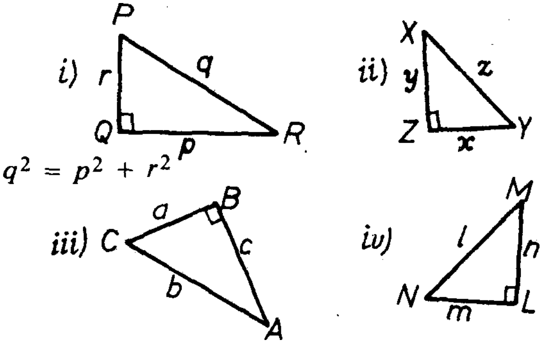
In the above form, the theorem of Pythagoras can be stated in words as follows:

> **'In a right angled triangle, the square of the hypotenuse is the sum of the squares of the other two sides.'**

Here, you must have understood that the square of the hypotenuse and the square of its sides means the square of their lengths.

We are going to study different proofs of this theorem later. First we shall study its application.

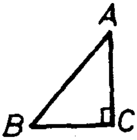
§ 1.4 In fig. 1.6, some right angled triangles are shown. As per the theorem of Pythagoras, write the relation between the sides of the right angled triangle. As an illustration the relation between the sides of the first triangle is given below.



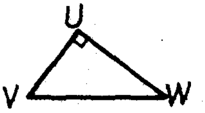
$q^2 = p^2 + r^2$

Fig. 1.6

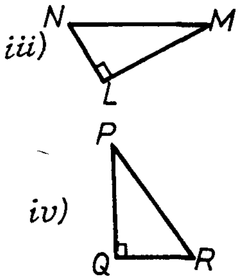
l(AB) means the length of the segment AB, and its square means $[\,l(\mathrm{AB})\,]^2$. Using this symbol express the theorem of Pythagoras for the given right angled triangles by filling up the blanks given below.

i) 

$[\,l(\mathrm{AB})\,]^2 = \square + \square$

ii) 

$[\,l(\mathrm{VW})\,]^2 = \square + \square$



$[l(MN)]^2 = \square + \square$

$[l(PR)]^2 = \square + \square$

Fig. 1.7

§ **1.5** If in Δ ABC, $m\angle C = 90°$ then, $c^2 = a^2 + b^2$ ....(i)
In this formula, the relation between three sides of a right angles triangle is given. Of the three sides, if the lengths of any two sides are known, the length of the third side can be determined by the use of the above formula.

Generally, the word side of the right angled triangle means the side other than the hypotenuse. The word hypotenuse of course means the side opposite to the right angle.

From the formula (i) we get,

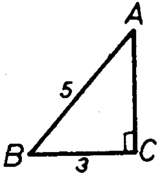
$$b^2 = c^2 - a^2 \text{ .... (ii)}$$
$$\text{and } a^2 = c^2 - b^2 \text{ .... (iii)}$$

In words the above formulae can be read as follows:

> **'In a right angled triangle, if we subtract the square of one side from the square of the hypotenuse, we get the square of the remaining side'**

From the data given in the following figures, find out answers to the questions asked.

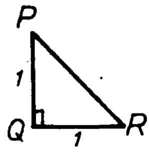
1



$l(AC) = ?$

Fig. 1.8

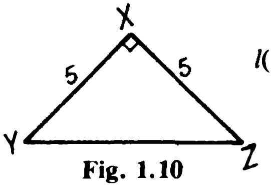
2



$l(PR) = ?$

Fig. 1.9

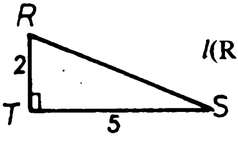
3



$l(YZ) = ?$

Fig. 1.10

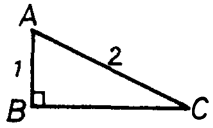
4



$l(RS) = ?$

Fig. 1.11

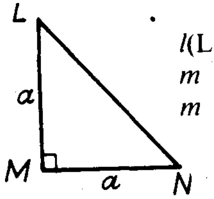
5



$l(BC) = ?$
$m \angle BAC = ?$
$m \angle BCA$?

Fig. 1.12

6



$l(LN) = ?$
$m \angle L = ?$
$m \angle N = ?$

Fig. 1.13

7

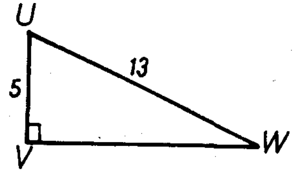


Fig. 1.14

$l(VW) = ?$

§ 1.6 Converse of the theorem of Pythagoras:

State the theorem of Pythagoras and its converse in terms of areas in 'when.....then' form.

State the theorem of Pythagoras in 'when...then' form in terms of the relation between the lengths of its sides.

Its converse will be as follows:

> **'If in a triangle, the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to the first side is a right angle'.**

Although the converse theorem can be proved, it is not our intention to prove this theorem at this stage.

Using the information given below, about the lengths of the sides of the triangle, state which angle of the triangle is a right angle.

(1) $a^2 = b^2 + c^2$. (2) $m^2 = l^2 + n^2$
(3) $[l(PR)]^2 = [l(PQ)]^2 + [l(QR)]^2$

In the following examples, small letters denote the lengths of the sides. State which triangles are right angled triangles. If a triangle is a right angled triangle, name the side opposite to the right angle.

(1) $a = 3, b = 5, c = 3$.
(2) $p = 5, q = 4, r = 3$
(3) $u = 8, v = 6, w = 10$.
(4) $l = 4, m = 9, n = 12$.
(5) $r = 5, s = 12, t = 13$.
(6) $l = 9, m = 41, n = 40$.
(7) $a = 30, b = 40, c = 60$.
(8) $p = 16, q = 30, r = 34$.
(9) $u = 10, v = 24, w = 26$.
(10) $m = 25, s = 24, t = 7$.

§ 1.7 Review of a few properties before proving the theorem of Pythagoras:

As a preparation for the proof of the theorem of Pythagoras, we shall first revise some important properties.

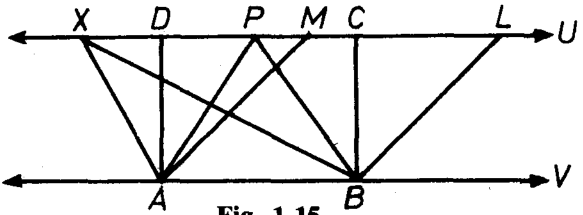


Fig. 1.15

In fig. 1.15 line U ∥ line V. There is a certain relation between the areas of the rectangle ABCD, the parallelogram ABLM, the △ABX, and △ABP. State this relation.

When a parallelogram and a triangle stand between the same two parallels and on the same base, then the area of the given triangle is half of the area of the given parallelogram.

State a similar property about two parallelograms.

With reference to the fig. 1.15, fill in the blanks in the following:—

1. (i) A(□ABCD) = A(□......). (ii) A(△APB) = ...A(□ABLM).
   (iii) .... = $\frac{1}{2}$ A(□ABCD).
2. If A(△APB) = 16, then (i) A(△ABX) = ...
   (ii) A(□ABCD) = ... (iii) A(□ABLM) = ...
3. If l(AB) = 5, l(AD) = 4, then
   (i) A(□ABLM) = ... (ii) A(△AXB) = ...

§ 1.8 Now consider, the outline of the proof of the theorem of Pythagoras.

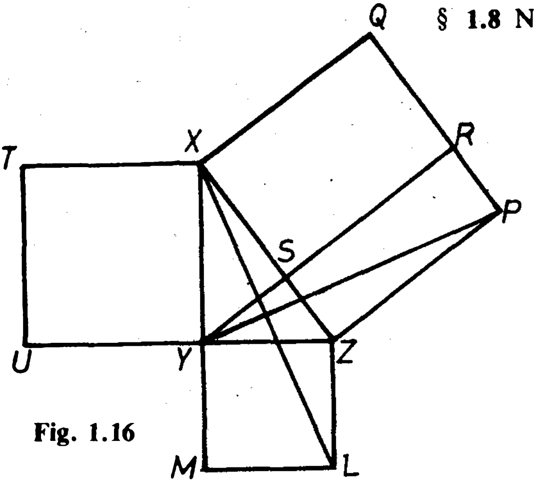


Fig. 1.16

In fig. 1.16, △XYZ is a right angled triangle. $m\angle XYZ = 90^\circ$. Square XZPQ on hypotenuse XZ and square YZLM on side YZ are constructed. They do not instersect the interiors of each other and the interior of △XYZ.

Now, answer the following questions:

1. Do, $\triangle$ ZLX and square ZLMY stand on the same base and are they between the same two parallels ? If so, name the parallel lines and the common base.
2. Are rectangle SZPR and $\triangle$ YZP between the same two parallels and stand on the same base ? If so, name the common base and the parallel lines.
3. To show that rectangle SZPR and square ZLMY are equal in area, is it sufficient to show that $\triangle$ ZLX and $\triangle$ YZP are equal in area ?
4. To show that $\triangle$ YZP and $\triangle$ ZLX are equal in area, is it sufficient to show that they are congruent ?
5. side ZP $\cong$ side XZ .....(why ?)
   side YZ $\cong$ side ZL .....(why ?)
6. Which angles are included by side XZ and side ZL, and also by side ZP and side YZ ? Which is the common angle in the interior of these angles ?

Answers to all the above questions lead us to conclude that

$\angle$ XZL $\cong$ $\angle$ YZP.

$\therefore$ By S-A-S test, $\triangle$XZL $\cong$ $\triangle$YZP

$\therefore$ A($\triangle$XZL) = A($\triangle$YZP)

$\therefore$ A(square YZLM) = A(Rectangle SZPR) ......(1)

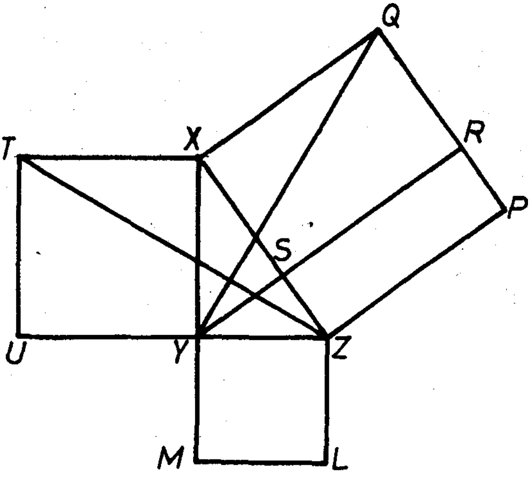


Fig. 1.17

Now, fig. 1.16 and fig. 1.17 are the same. In fig. 1.17, seg TZ instead of seg XL and seg YQ instead of seg YP are joined.

From fig. 1.17 you will easily notice the following,

$\triangle$XTZ $\cong$ $\triangle$XYQ ....(S-A-S test)

$\therefore$ A($\triangle$XTZ) = A($\triangle$XYQ)

$\therefore$ A(square TXYU) = A(Rectangle XSRQ) ....(2)

From (1) and (2) we get,
A(square TXYU) + A(square YZLM) = A(square XZPQ).

(2)

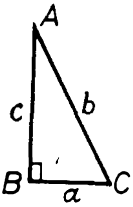


Fig. 1.18

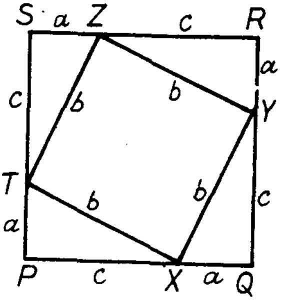


Fig. 1.19

In the fig. 1.19, four right angled triangles congruent to △ABC are placed in such a position that square PQRS, the length of whose side is $(a + c)$, is formed. In fig. 1.19 it is easily seen that ☐XYZT is a rhombus, but to show that it is also a square, we will have to show that one of its angles is a right angle. (Try to show this)

A(square PQRS) = A(square XYZT) + 4 A(△ABC)

$\therefore\ (a+c)^2 = b^2 + 4 \times \frac{1}{2} a \times c$

$\therefore\ a^2 + 2ac + c^2 = b^2 + 2ac$

$\therefore\ a^2 + c^2 = b^2$

This proves the theorem of Pythagoras.

(3)

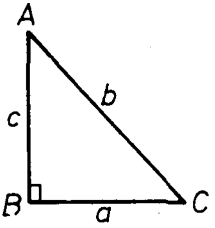


Fig. 1.20

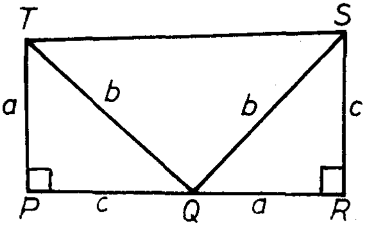


Fig. 1.21

In fig. 1.21 two right angled triangles congruent to △ABC, are placed in such a position that P-Q-R and $l(PR) = a + c$. In fig. 1.21 prove that △TQS is a right angled triangle, so that $m\angle TQS = 90°$

Now,

A(trapezium PRST) = A(△TQS) + 2 A(△ABC)

$\therefore\ \frac{1}{2}(a + c)(a + c) = \frac{1}{2}b^2 + 2 \times \frac{1}{2} a \times c$

$\therefore\ a^2 + 2ac + c^2 = b^2 + 2ac$

$\therefore\ a^2 + c^2 = b^2$

This is yet another proof of the theorem of Pythagoras.

§ 1.9 Solved Examples:

1. Two parallel poles, perpendicular to the leveled ground, are each 50 *m* and 55 *m* high. They are 12 *m* apart. Find the distance between their tops.

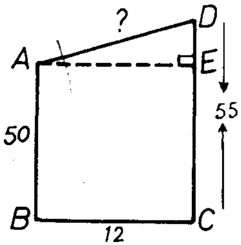


Fig. 1.22

Given: $l(AB) = 50$, $l(DC) = 55$ and $l(BC) = 12$

Now, $l(DE) = l(DC) - l(EC) = 5$

$l(BC) = l(AE) = 12$

$\triangle AED$ is a right angled triangle.

$\therefore$ By Pythagoras Theorem,

$$[l(AD)]^2 = [l(AE)]^2 + [l(DE)]^2$$

$$\therefore [l(AD)]^2 = (12)^2 + (5)^2$$

$$= 144 + 25$$

$$= 169$$

$$\therefore l(AD) = 13$$

$\therefore$ The distance between the tops of the poles is 13 *m*.

2. Two buildings are opposite to each other. Their walls are parallel. A ladder 150 *dm* long is placed so that its top reaches 144 *dm* above the road. On turning the ladder over to the other side of the street, its top reaches a point 42 *dm* above the road. Find the breadth of the street.

Given: seg PB is the original position of the ladder and seg PD is the later position of the ladder.

$l(AB) = 144$, $l(PB) = l(PD) = 150$ and $l(DC) = 42$

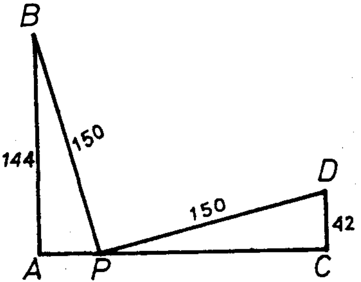


Fig. 1.23

In $\triangle APB$,

$[l(BP)]^2 = [l(AB)]^2 + [l(AP)]^2$ ......(Theorem of Pytha.)

$\therefore (150)^2 = (144)^2 + [l(AP)]^2$

$\therefore (150)^2 - (144)^2 = [l(AP)]^2$

$\therefore 42 = l(AP)$

Now, in $\triangle PCD$,

$[l(PD)]^2 = [l(DC)]^2 + [l(PC)]^2$ ......(Theorem of Pytha.)

$\therefore (150)^2 = (42)^2 + [l(PC)]^2$

$\therefore (150)^2 - (42)^2 = [l(PC)]^2$

$\therefore 144 = l(PC)$

$\therefore$ Width of the road $= l(AC)$

$= l(AP) + l(PC)$

$= 42 + 144 = 186$ *dm*

Miscellaneous Exercise - 1

1. The length of the side of a square is 10 cm. Find the length of its diagonal.
2. The length of a diagonal of a square is 16 cm. Find the length of its side.
3. The length of a side of an equilateral triangle is 8 cm. Find its height.
4. The length of a side of a square is a. Find the length of its diagonal.
5. The length of a side of an equilateral triangle is a. Find its height and area.
6. The length of a diagonal of a square is $10\sqrt{2}$. Find its perimeter.

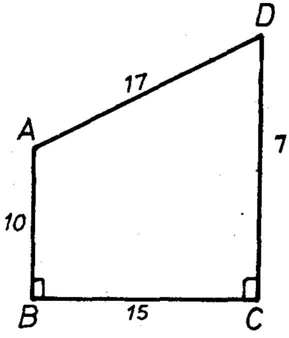


Fig. 1.24

7. In fig. 1.24, if $l(\mathrm{AB}) = 10$, $l(\mathrm{BC}) = 15$ and $l(\mathrm{AD}) = 17$, find $l(\mathrm{DC})$.

8. In fig. 1.25 □ABCD is a trapezium with seg AB || seg DC, $m\angle \mathrm{DAB} = 45^\circ$, $m\angle \mathrm{CBA} = 30^\circ$, $l(\mathrm{DC}) = 4$ and $l(\mathrm{BC}) = 10$. Find $l(\mathrm{AB})$ and the area of □ABCD.

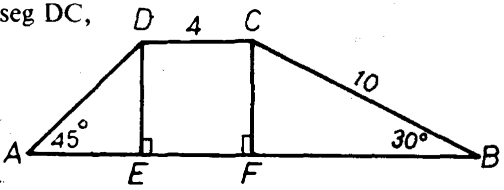


Fig. 1.25

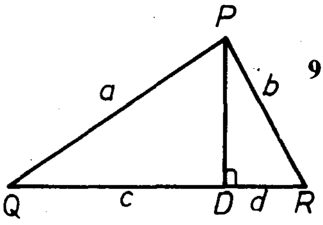


Fig. 1.26

9. In $\triangle$PQR, seg PD $\perp$ seg QR. $l(\mathrm{PQ}) = a$, $l(\mathrm{PR}) = b$, $l(\mathrm{QD}) = c$ and $l(\mathrm{DR}) = d$. Prove that, $(a + b)(a - b) = (c + d)(c - d)$

10. In fig. 1.26, if $a = 9$, $b = 7$, $c = 6$ then $d = ?$ and A($\triangle$PQR) = ?

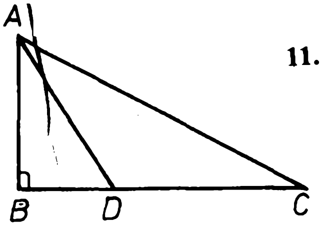


Fig. 1.27

11. In fig. 1.27, $m \angle ABC = 90°$ and B-D-C. Prove that
$[l(BC)]^2 + [l(AD)]^2$
$= [l(AC)]^2 + [l(BD)]^2$

12. In $\triangle ABC$, $m \angle C = 90°$, $l(AB) = c$, $l(BC) = a$ and $l(CA) = b$. Prove that ka, kb, kc are also the sides of a right angled triangle.
[If three numbers are related by Pythagoras theorem, then the three numbers are known as a triplet of Pythagoras. Since, $5^2 = 4^2 + 3^2$, we have 3, 4, 5 as a triplet of Pythagoras.]

13.

No.	Length of the side		Hypotenuse
	1st	2nd	
1	3	4	5
	2	8	8
2	5	12	13
	2	12	12
3	7	24	25
	2	16	16
4	9	40	41
5	—	—	—
6	—	—	—
7	—	—	—

In the table some triplets of Pythagoras are given. Study the table and find out the remaining triplets.

The triplets of Pythagoras can be obtained from the following tricky expression, which is to be interpreted as a vulgar fraction, $n\frac{n}{2n+1}$. In this expression if n is a natural number, we get the lengths of the three sides of a right angled triangle as follows:

Putting $n = 1$, $n\frac{n}{2n+1} = 1\frac{1}{3} = \frac{4}{3}$. The first number 3 appears in the denominator, the second number is 4 in the numerator. The third number 5 (hypotenuse) is obtained by adding 1 to the number in the numerator i.e. $4 + 1 = 5$.

Putting $n = 2$, $n\frac{n}{2n+1} = 2\frac{2}{5} = \frac{12}{5}$. Then the triplet of Pythagoras is (5, 12, 13)

14. Application of the theorem of Pythagoras.

(a)

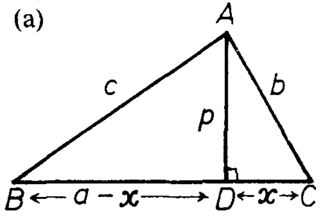


Fig. 1.28

In $\triangle ABC$, $\angle C$ is an acute angle, seg AD $\perp$ seg BC. Small letters in the fig. 1.28 denote the lengths of the sides. Using the theorem of Pythagoras, fill in the blanks given below.

In $\triangle ADB$,

$c^2 = (a - x)^2 + \boxed{p^2}$

$\therefore c^2 = a^2 - 2ax + x^2 + \boxed{p^2}$ ...(1)

In $\triangle ADC$,

$b^2 = x^2 + \boxed{p^2}$ ....(2)

From (1) and (2) we get,

$c^2 = a^2 - 2ax + \boxed{b^2}$ .....(3)

$\therefore [l(AB)]^2 = [l(BC)]^2 + [l(AC)]^2 - 2[l(BC) \times l(DC)]$

(b) $\triangle$ ABC is an obtuse angled triangle, with $\angle$ ACB as obtuse. Seg AD $\perp$ line BC. Small letters denote the lengths of the sides.

Fill in the blanks by applying the theorem of Pythagoras to right angled triangles in fig. 1.29.

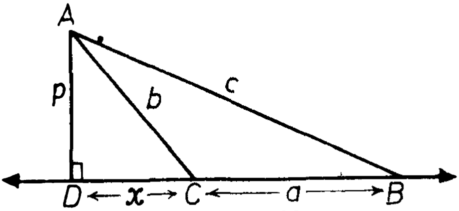


Fig. 1.29

In $\triangle ADB$,

$c^2 = (a + x)^2 + \boxed{\quad\quad}$

$\therefore c^2 = a^2 + 2ax + x^2 + \boxed{\quad\quad}$ ....(1)

In $\triangle ADC$

$b^2 = x^2 + \boxed{\quad\quad}$ ....(2)

From (1) and (2)

$c^2 = a^2 + 2ax + \boxed{\quad\quad}$ ....(3)

$\therefore [l(AB)]^2 = [l(BC)]^2 + [l(AC)]^2 + 2[l(BC) \times l(CD)]$

15. Theorem of Appollonius:

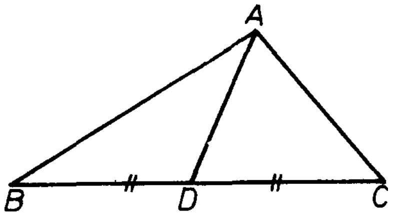


Fig. 1.30

In $\triangle ABC$, point D is the mid-point of side BC. We, then, have the following property,

$[l(AB)]^2 + [l(AC)]^2$

$2[l(AD)]^2 + 2[l(DC)]^2$

This is known as the theorem of Appollonius. Solve the example below as an application of this theorem.

If in △ABC, l(AB) = 4, l(AC) = 6, l(DC) = 4 then, find l(AD).

16. In △PQR, *m* ∠ QRP = 90° seg RT ⊥ seg QP, *l*(QR) = 5, *l*(RP) = 12. Find (i) *l*(QP), (ii) *l*(RT).

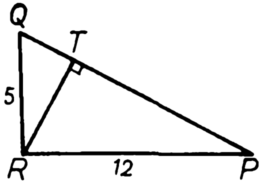


Fig. 1.31

17. In△ PQR, point S is the mid point of the side QR. *l*(QP) = 11, *l*(PR) = 17, *l*(PS) = 13. Find *l*(QR).
18. In △ABC, *m* ∠ B = 90°, *l*(AB) = 6, *l*(BC) = 8. What is the length of the median drawn from the point B on the side AC ?
19. In a right angled triangle the length of the hypotenuse is 65 cm. and the length of one of the perpendicular sides is 16 cm. Find the area of the triangle.
20. Use the fact $(5)^2 + (2)^2 = 29$, to construct a segment of length $\sqrt{29}$.
21. □ ABCD is a trapezium in which seg AB ∥ seg DC, seg DE ⊥ seg AB, A-E-B. *l*(AD) = 10, *l*(DC) = 12, *l*(CB) = 17, *l*(AE) = 6. Then find A(□ABCD).
22. In △ABC, *l*(AB) = 10, *l*(AC) = 7 and *l*(BC) = 5, Find the length of the median drawn from the point C to the side AB.
23. In △PQR, side PS is the median. If *l*(PS) = 4 and $[l(PQ)]^2 + [l(PR)]^2 = 130$. Find *l*(QR).
24. The length of each side of a rhombus is 10 cm and the length of one of its diagonals is 12 cm. What is the area of the rhombus ?
25. In a trapezium ABCD, seg AB ∥ seg DC. seg BD ⊥ ~~seg~~ AD. seg AC ⊥ seg BC. If *l*(AD) = 15, *l*(BC) = 15, *l*(AB) = 25 then find A(□ABCD).

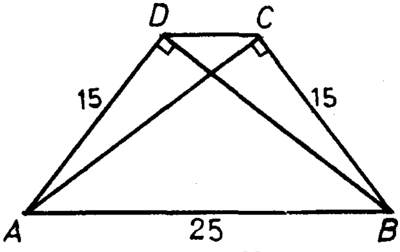


Fig. 1.32

26. If the diagonals of a quadrilateral intersect at right angles, prove that the sum of the squares of a pair of opposite sides is equal to the sum of the squares of the other pair.

□ □ □

Chapter 2

CIRCLE

§ 2.1 Circle-radius-centre.

You know how to draw a circle. So, draw a neat circle on the plane of a paper. Now answer the following:

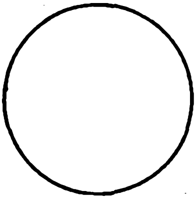


Fig. 2.1

While you were drawing the circle,
(i) did the distance between the two ends of your compass change ? [Ans : No]
(ii) Of the two ends of your compass did some end remain fixed ? [Ans : Yes]
Which one ? [Ans : the end without pencil]

Look at the figure you have drawn, observe that
(i) The figure of a circle is a closed figure.
(ii) The entire figure is in the plane of the paper
(iii) The point at which one end of your compass remained fixed is such that any point of the figure is at the same distance from this point.

There is no other point in the plane of the figure which has this property.

So,

'A circle is a set of points in a plane such that any point of this set is at a fixed distance from a fixed point of the plane.'

'Centre of a circle is a fixed point in the plane of the circle such that any point of the circle is at a fixed distance from it.'

'Radius of a circle is the distance between the centre of that circle and any point of it.'

A circle in a plane is determined if its centre and radius is known.

Sometimes however the word radius is used in a sense different from the one given above.

A segment joining the centre of a circle to any point on the circle is also called a radius of the circle.

Mostly the context in which the term is used makes its sense clear.

§ 2.2 Some explanation about terms:

A point of a circle is said to lie on that circle. We also say that a circle passes through a point which lies on it.

The end point of a radius of a circle which lies on the circle is said to be the outer end of that radius.

In the given fig. (2.2), P is the centre of a circle. Points P and R appear to be inside the circle, Q appears to be on it, whereas S appears to be outside the circle.

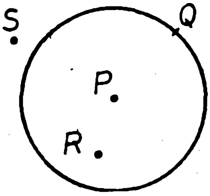


Fig. 2.2

If you now measure $d(P,R)$ it will be less than the radius of the circle. Similarly, if you measure $d(P,S)$ it will be greater than the radius of the circle. Thus for any point R of the plane if $d(P,R) <$ radius, R will be inside the circle. Similarly for any point S of the plane if $d(P,S) >$ radius, S will be outside the circle.

When R is a point in the plane of a circle such that its distance from its centre is less than its radius, R is said to be inside the circle, or to be in the interior of the circle.

When R is a point in the plane of a circle such that its distance from the centre is greater than its radius, R is said to be outside the circle or to be in the exterior of the circle.

A point of a circle is neither in its inside nor in its outside.

Because of a circle, the set of points in the plane of the circle is divided into three disjoint sets as follows:

(i) set of points lying inside a circle.
(ii) set of points lying on the circle.
(iii) set of points lying outside the circle.

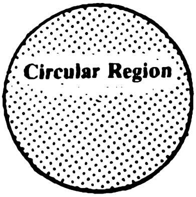


Fig. 2.3

The set of points lying on a circle together with the set of points lying inside it form a circular region or a disc.

The distinction between the meanings of the terms (i) a circle (ii) a circular region should clearly be understood.

§ 2.3 Circles passing through given points:

Given a set of points in a plane we wish to decide whether a circle can be drawn in this plane passing through the given points. If the answer is yes we also wish to decide on the number of such circles. We will restrict our discussion to the following cases.

(*a*) **Given a point P in a plane.** Consider a point Q in the same plane other than P. We see that with Q as centre and *d*(P,Q) as radius if a circle is drawn it will pass through P. Since this is true for every point in the plane, other than P, it is clear that infinite number of circles can be drawn passing through a given point of a plane.

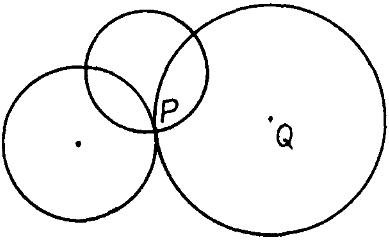


Fig. 2.4

(*b*) **Given two points P & Q in the plane.**

We know that all points on the perpendicular bisector of seg PQ are equidistant from P and Q. So with any point R on the perpendicular bisector of seg PQ as centre and *d*(P,R) or *d*(Q,R) as radius if a circle is drawn it will pass through both points P and Q. From this it is clear that infinite number of circles can be drawn passing through two given points in a plane.

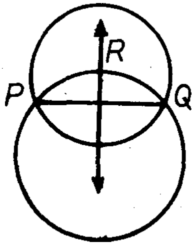


Fig. 2.5

(*c*) **Given three collinear points P, Q and R.**

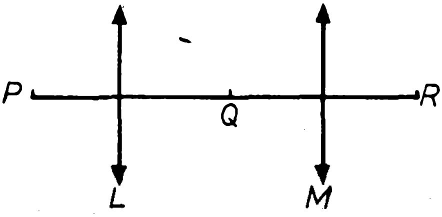


Fig. 2.6

We know that any point on the perpendicular bisector of seg PQ is a centre of a circle which will pass through points P & Q. Similarly any point on the perpendicular bisector of seg QR is a centre of a circle which will pass through the points Q and R. Let line L be the perpendicular bisector of seg PQ and line M be the perpendicular bisector of seg QR. Since points P,Q and R are collinear seg PQ and seg QR are segments of the same line. Line L and line M are perpendicular to the same line on which given points P, Q and R lie. Therefore line L is parallel to line M. Being parallel these lines do not have a common point. Thus we can not find a point which will serve as a centre of a circle which will pass through points P, Q and R.

There is no circle which passes through three given collinear points.

(*d*) **Given three non-collinear points A, B and C.**

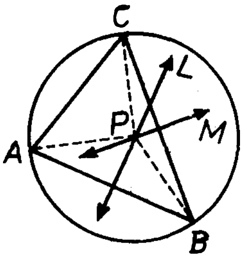


Fig. 2.7

The centre of a circle passing through the points A, B and C which are non-collinear would lie on perpendicular bisector L of seg AB and perpendicular bisector M of seg BC. It can be shown that lines L and M are non-parallel. Hence, they intersect in exactly one point. This point obviously has the property that it is equidistant from points A, B and C. If this point is named as P we see that a circle drawn with P as centre and $d(P,A) = d(P,B) = d(P,C)$ as radius, this circle has to pass through the points A, B and C. So, one and only one circle can be drawn passing through these given non-collinear points.

In other words,

Given three non-collinear points there is one and only one circle which passes through these points.

We know that the vertices of a triangle are three non-collinear points. So we can always have a circle passing through the vertices of a triangle. In other words with every triangle we can associate a circle which passes through its vertices. Such a circle is called the **circumcircle** of a given triangle and its centre is called **circumcentre.**

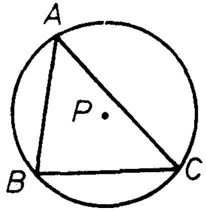


Fig. 2.8

(*e*) **Given four points in a plane.**

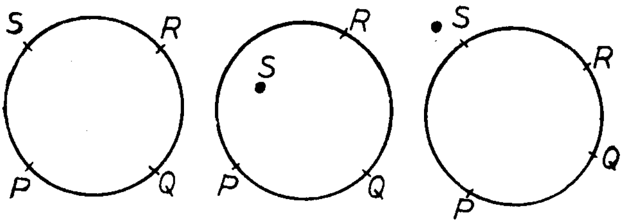


Fig. 2.9

It is clear that if any three of these four points are collinear no circle can be drawn which will pass through these points. When no three of these four points are collinear,only in special circumstances we can draw a circle which will pass through all the four points.

When four or more points lie on the same circle they are said to be **concyclic.**

Exercise 2.1

1. State which of the following statements are true or false. Correct those statements which are false.
   (*a*) A circle is a set of points at a fixed distance from a fixed point.
   (*b*) A point of a circle is inside a circle.
   (*c*) There are only a finite number of points inside a circle.
2. Explain why the following figures are not the figures of a circle:

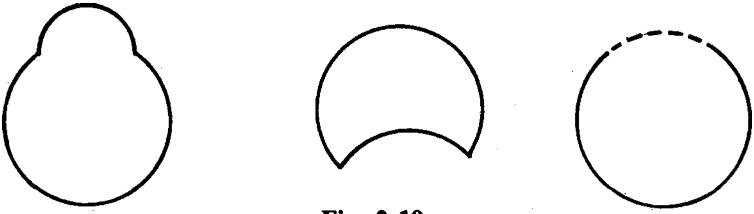


Fig. 2.10

3. Given a circle C with the centre P and radius r, if P-Q-R and R is inside a circle, show that Q is also in its inside.
4. Given a circle C with centre O and a point A on it. If A-O-B and B is outside the circle show that $d(A,B) > 2$ (radius of C).
5. Given a circle C with centre O and radius r and points A and B inside the circle C. Show $l(AB) < 2r$.
   (Hint : Use triangle inequality)
6. In the given figure if radius $= r$ show $l(AB) < 2r$. When will $l(AB)$ be equal to $2r$? In the same figure if A-C-B, show that C is inside the circle.

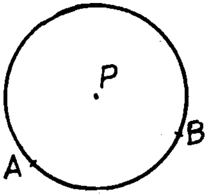


Fig. 2.11

7. Given a line L and points A, B as in fig. 2.12. Find a point P on L such that $l(PA) = l(PB)$.

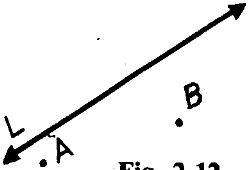


Fig. 2.12

8. Given a line L and points A and B as in fig. 2.13. Find a point P on L such that $l(PA) = l(PB)$.

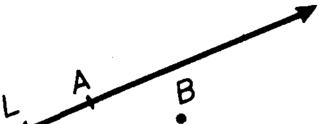


Fig. 2.13

9. Lines L and M intersect in point Q as shown in the figure. P is on the bisector of an angle at Q. Find a point A on L and a point B on M such that $l(PA) = l(PB)$ and $l(QA) = l(QB)$.

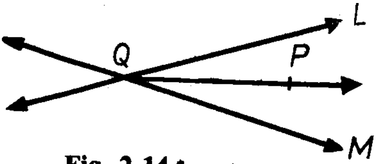


Fig. 2.14

10. Given three non-collinear points A, B and C as in fig. 2.15. State how you will find a point P such that $l(PA) = l(PB) = l(PC)$.

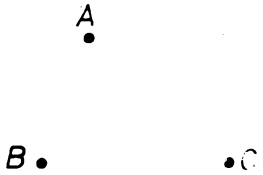


Fig. 2.15

§ 2.4 Some more terms:

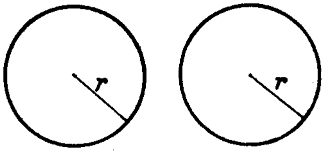


Fig. 2.16 (*a*)

(*a*) The term congruent circles means circles having the same radius.

(*b*) Concentric circles means circles having the same centre but different radii.

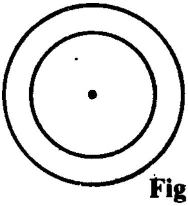


Fig. 16 (*b*)

(*c*) Chord of a circle means a segment whose end points are on the given circle. In fig. 2.16 seg AB, seg CD are chords of the given circle whereas seg PB, seg EF and seg GH are not chords of the given circle.

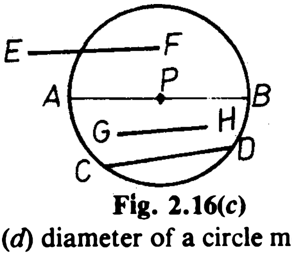


Fig. 2.16(*c*)

(*d*) diameter of a circle means a chord of a circle which passes through its centre. In fig. 2.16 (c) chord AB is a diameter where as chord CD is not a diameter.

§ **2.5** Observe that a circle seems to be symmetrically placed about its diameter. A line containing a diameter of a circle is called an axis of symmetry of that circle. In your previous years you have seen how on the basis of symmetry many properties about a circle could be infered. We shall study those properties and some more in the next few articles.

§ 2.6 Theorem:

The line joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

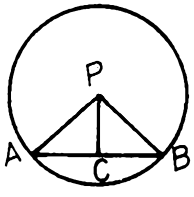


Fig. 2.17

Given

(1) A circle with centre P and radius r.

(2) A chord AB.

(3) C the mid-point of the chord AB
i.e. seg CA ≅ seg CB.

To prove: seg PC ⊥ chord AB.

Proof: Consider △PCA and △PCB. We have,

seg PA ≅ seg PB (radii of the same circle)

seg AC ≅ seg CB (Given), seg PC ≅ seg PC

∴ △PCA ≅ △PCB (SSS congruence)

∠PCA ≅ ∠PCB (corresponding angles of congruent triangles)

But, $m\angle PCA + m\angle PCB = 180°$ (Angles of linear pair)

∴ $2m\angle PCA = 180°$

∴ $m\angle PCA = m\angle PCB = 90°$

∴ seg PC ⊥ chord AB.

We may prove the above theorem also as follows

seg AP ≅ seg BP (radii of the same circle)

seg AC ≅ seg BC (Given)

So points P and C are both equidistant from points A & B.

∴ Points P and C lie on the perpendicular bisector of seg AB.

∴ seg PC ⊥ chord AB.

§ 2.7 The converse of the theorem in § 2.6 is also true. Below we state the converse and prove it.

Theorem: The perpendicular from the centre of a circle to its chord bisects the chord.

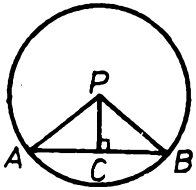


Fig. 2.18

Given: 1. A circle with centre P and radius r

2. A chord AB

3. seg PC ⊥ chord AB
where C is on chord AB
i.e. ∠PCA ≅ ∠PCB

To prove: C is the midpoint of the chord i.e. seg AC ≅ seg CB.

Proof: In right angled triangles △PCA and △PCB

seg PA ≅ seg PB (radii of the same circle)

seg PC ≅ seg PC (Given)

△PCA ≅ △PCB (Hypo.—side theorem)

∴ seg CA ≅ seg CB (corresponding sides of congruent triangles.)

Exercise — 2.2

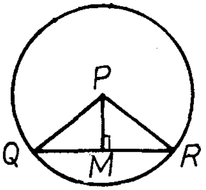


Fig. 2.19

1. **Given**: radius PQ = 10
   l(QR) = 12
   Find : *l*(PM)

2. **Given**: radius = 6
   l(AD) = 3
   Find : *l*(BC)

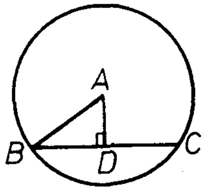


Fig. 2.20

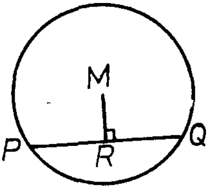


Fig. 2.21

3. **Given**: centre M
   l(MR) = 8
   l(PQ) = 8
   Find: radius

4. **Given**: centre O
   m ∠ AOB = 60°
   l(AO) = 4
   Find: distance of chord AB from O.

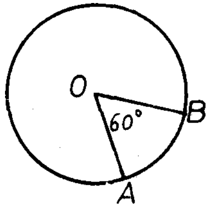


Fig. 2.22

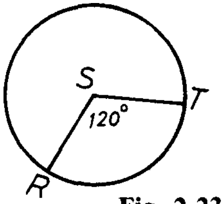


Fig. 2.23

5. **Given**: centre S
   m ∠ RST = 120°, radius = 10
   Find: (i) *l*(RT)
   (ii) distance of S from seg RT.

6. Given a chord EF of length 8 of circle with centre D and radius 5. Find the distance of the chord from the centre.

7. Given a chord AB of length 10 of a circle with centre O and *m* ∠ OAB = 45°. Find the radius of the circle.

8. In the adjoining fig. C_1 and C_2 are congruent circles. Chord AB of C_1 ≅ chord CD of C_2. Point P is the centre of C_1. Point Q is the centre of C_2. seg PM ⊥ chord AB with M on chord AB. seg QN ⊥ chord CD with N on chord CD.

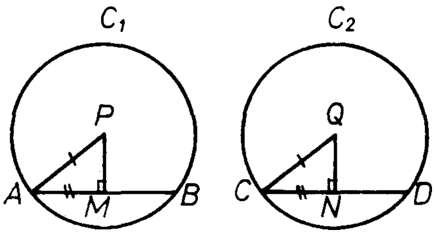


Fig. 2.24

Aim: to prove that seg PM ≅ seg QN.
Hint: Use Hypotenuse side theorem to show first that Δ APM ≅ Δ CQN
[Congruent chords of congruent circles are equidistant from their respective centres.]

9. In the adjoining fig. C_1 is a circle with centre P. Chord AB ≅ chord CD.
   seg PM ⊥ chord AB with M on chord AB.
   seg PN ⊥ chord CD with N on chord CD.

Aim: to prove that seg PM ≅ seg PN
[Congruent chords of the same circle are equidistant from the centre.]

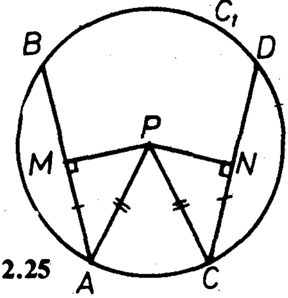


Fig. 2.25

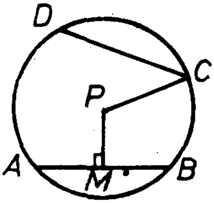


Fig. 2.26

10. **Given**: centre P
seg AB ≅ seg CD
seg PM ⊥ seg AB
l(PM) = 4, l(PC) = 5
Find : l(CD)

Supplying the proof by filling the blanks.

By observing the given diagram and after reading carefully the information supplied, you know how to fill in the blanks and complete the steps in the proof of a given geometrical property. Below are some examples in which you are required to fill in the blanks properly to complete the given proof.

11. **Given**: circle with centre O
seg PR ≅ seg PS
seg OM ⊥ chord PS, P-M-S
seg ON ⊥ chord PR, P-N-R

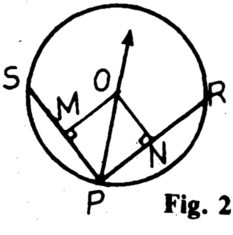


Fig. 2.27

To prove: ∠ OPM ≅ ∠ OPN.
Proof: Consider Δ OPM and Δ OPN,
seg OM ≅ seg _ _ _ _ _ (congruent chords are equidistant from the centre.)
seg PM ≅ seg _ _ _ _ _ (Perpendicular from the centre to the chord bisects the chord.)
seg OP ≅ _ _ _ _ _ _ _
∴ ΔOPM ≅ ΔOPN
∴ ∠ OPM ≅ ∠ _ _ _ _ (corresponding angles of congruent triangles)

(If congruent chords of a circle intersect on a point of the circle the centre of the circle lies on the bisector of the angle made by the chords.)

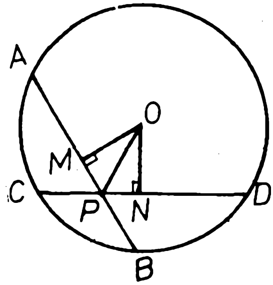


Fig. 2.28

12. Given: In the adjoining figure of a circle with centre O, chord AB and chord CD intersect at a point P inside the circle and chord AB $\cong$ chord CD.

seg OM $\perp$ chord AB, A-M-B.

seg ON $\perp$ chord CD, C-N-D.

To prove: 1. seg PA $\cong$ seg PD.

2. seg PB $\cong$ seg PC

3. $\angle$ OPA $\cong$ $\angle$ OPD

Proof: In OPM $\longleftrightarrow$ OPN

seg OM $\cong$ seg___ (congruent chords are equidistant from the centre)

seg OP $\cong$ seg _ _ _

$\triangle$OPM $\cong$ $\triangle$ OPN (Hypo-side theorem)

$\therefore$ seg PM $\cong$ seg______ (corresponding sides of congruent triangles)

Now seg MA $\cong$ seg ND (bisected segments of congruent segments)

l(PM) + l(MA) = l(_ _) + l(ND)

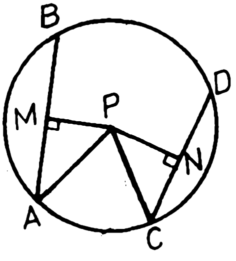
$\therefore$ l(PA) = l(PD) (P-M-A and _ _ _ _)

Also, l(AB) = l(CD) ——given

So, l(AB) − l(PA) = l(CD) − l(PD)

Also, $\angle$ OPA $\cong$ $\angle$_____ (corresponding angles of congruent triangles)

[If two congruent chords of a circle intersect inside a circle parts of one chord made by the point of intersection are congruent respectively to parts of the other. The centre of the circle lies on the bisector of that angle made by the chords which contains the centre in its interior.]



13. Radius of the circle with centre P is 6.
l(chord AB) = 8, l(chord CD) = 6.
seg PM $\perp$ chord AB, A-M-B.
seg PN $\perp$ chord CD, C-N-D.
find l(PM), l(PN). Compare their lengths.

Fig. 2.29

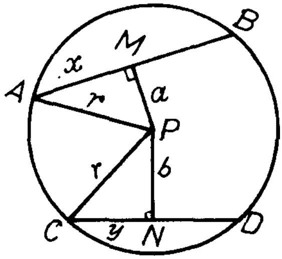


Fig. 2.30

14. **Given**: In a circle with centre P, l(chord AB) = $2x$, l(chord CD) = $2y$ and $x > y$
[i.e. chord AB > chord CD]
seg PM $\perp$ chord AB, A-M-B.
seg PN $\perp$ chord CD, C-N-D.
l(PM) = a, l(PN) = b, l(AP) = l(CP) = r.

To prove: $a < b$.

Proof: from right angled triangles, $\triangle$ APM & $\triangle$ PCN we have

$r^2 = a^2 + x^2 = b^2 + y^2$

$\therefore\ x^2 - y^2 = b^2 - \boxed{}$ I

But $x > \boxed{}$ (Given)

$\therefore\ x^2 > \boxed{}$

$\therefore\ x^2 - \boxed{} > 0$

$\therefore\ b^2 - \boxed{} > 0$

$\therefore\ b > \boxed{}$ (from I)

$\therefore\ a < b$.

[If two chords of a circle are unequal in length, larger chord is nearer to the the centre than the smaller one.]

15. Use ex. (14) to show that diameter of a circle is the largest of all the chords of the circle.

16. **Given**: Two concentric circles with centre P have A, B, C, D as the points of intersection with line L as shown in fig. l(AD) = 12, l(BC) = 8.
To prove: l(AB) = l(CD) = 2
l(AC) = l(BD) = 10.
(**Hint**: Let seg PM be perpendicular to line L with M on L and use the property that the perpendicular from the centre bisects the chord.)

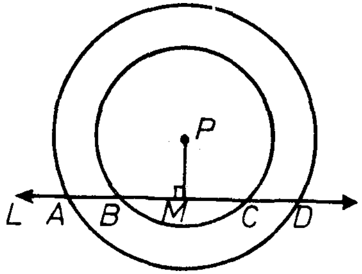


Fig. 2.31

17. The lengths of the adjacent sides of a rectangle ABCD are 3 cm and 4 cm respectively. Find the radius of the circle passing through its vertices.

§ 2.8 Semicircle

A line containing the diameter of a circle as we know is the axis of symmetry for the circle. Points of the circle lying on one side of the axis together with end points of the diameter are said to be on a semicircle.

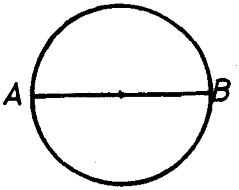


Fig. 2.32

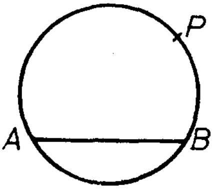


Fig. 2.34

If seg AB is a chord of a circle on which P is any point other than A and B, we say that chord AB subtends ∠ APB at P. ∠ APB is said to be subtended by the chord AB.

In the adjoining figure of a circle with O as centre seg AB is a diameter. P is any point on the circle. Measures of angles are shown by letters. Since sum of the measures of angles of △ APB is 180° we have,

$$x + y + (x + y) = 180^\circ$$

$$\therefore \quad 2(x + y) = 180^\circ$$

$$\therefore \quad x + y = 90^\circ$$

$$\therefore \; m\angle APB = 90^\circ$$

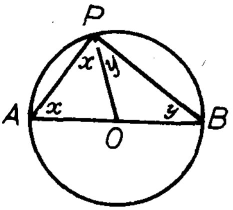


Fig. 2.34

We have the following very important property:

> "Diameter of a circle subtends a right angle at a point on the circle."

When expressed in 'when-then' form the above property reads as,

> "When a diameter of a circle subtends an angle at a point on the circle then that angle is a right angle."

Now, it is possible for us to write the converse of the above statement which is as follows.

When a segment subtends a right angle at a point, the point lies on the circle with given segment as diameter.

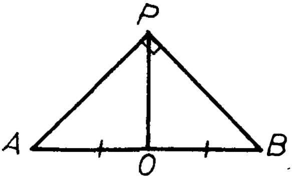


Fig. 2.35

You know that the median to the hypotenuse of a right angled triangle has half the length of the hypotenuse.

Therefore, when a segment AB subtends a right angle at P and O is the midpoint of seg AB we have

$$l(OP) = l(OA) = l(OB)$$

So,

With centre O and l(OA) as radius if a circle is drawn this circle will pass through P.

Now that we know that a diameter of a circle subtends a right angle at any point of it, it would be interesting to find out as to what type of angle a diameter will subtend at a point when this point is outside or inside the circle.

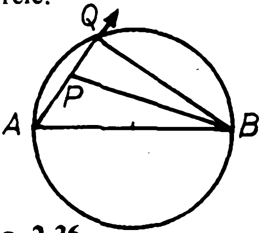


Fig. 2.36

In fig. 2.36 chord AB is a diameter of the circle. P is any point inside the circle. Ray AP intersects the circle in Q. $m \angle AQB = 90°$. By exterior angle theorem of a triangle,

$m \angle APB > m \angle AQB$

$m \angle APB > 90°$

$\therefore \angle APB$ is obtuse.

Hence a diameter of a circle subtends an obtuse angle at a point inside a circle.

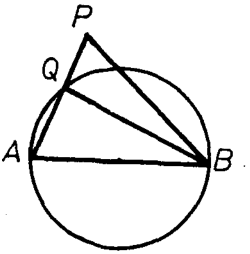


Fig. 2.37

In the adjoining fig.(2.37) chord AB is a diamter of a circle and point P is outside a circle. Let side AB intersect the circle in Q. Join Q to B. Again applying exterior angle theorem to $\triangle BQP$ we have

$m \angle AQB > m \angle QPB$

$\therefore \quad 90° > m \angle APB$

Diameter of a circle subtends an acute angle at a point P outside the circle.

§ 2.9 About a secant and tangent:

We now study a circle in relation to lines in its plane.

Given a circle C and a line L in the plane of the circle we see that three types of situations arise as shown in the figures below.

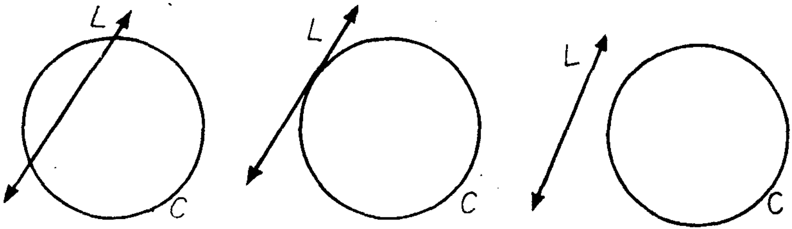


Fig. 2.38 (i) L intersects C in two points. (ii) L intersects C in one point. (iii) L intersects C in no point

Since, no circle passes through three collinear points no line will intersect a circle in more than two point. When a line intersects a circle in two distinct points it is said to be a SECANT of the circle.

When a line intersects a circle in one and only one point it is said to be a TANGENT-LINE to the circle. It is also said to be tangent to the circle.

From the figure we observe the following:

When a line is a secant of a circle it has some points inside the circle, whereas when a line is tangent to the circle it has no points inside the circle. In fact the following statement can be proved, although we will not be interested in its proof at this stage.

> "When one point of a given line in the plane of a circle is inside the circle, then the line intersects the circle in exactly two points."

When a line intersects a circle in one and only one point the point is called a **point of contact**. The point of contact is the only point which is common to the tangent line and the circle. This also means that every point of a tangent line to a circle except the point of contact lies outside the circle. Since, every point outside a circle is from its centre at a distance greater than the radius of the circle, it is clear that of all the points of a tangent line to a circle the point of contact is the nearest to the centre of the circle. This discussion leads to the following theorem.

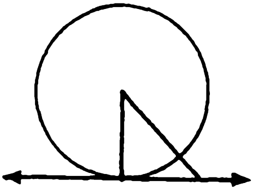


Fig. 2.39

§ 2.10 Theorem:

A line perpendicular to a radius of a circle at its outer end is tangent to the circle.

Given :

1) A circle C with centre P and radius r.
2) A point Q on C.
3) A line T perpendicular to radius PQ at Q.

To prove: The line T is tangent to the circle C at Q.

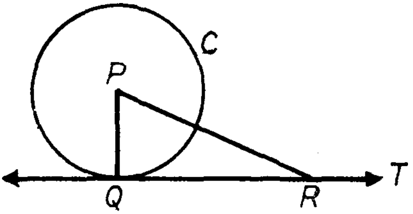


Fig. 2.40

Proof: Take a point R other than Q on the line T. Since seg PQ is $\perp$ to line T it is the shortest segment from P to the line T.

$\therefore\ l(PR) > l(PQ)$

$\therefore\ l(PR) > r.$

This shows that the point R is outside the circle.

This is true for every point of the line T other than the point Q.

$\therefore$ Q is the one and only one point which is common to line T and the Circle C.

$\therefore$ Line T is tangent to the circle C.

§ 2.11 Converse of the above theorem is as follows:

Theorem: When a line is a tangent line of a circle then that line is perpendicular to the radius of the circle at its point of contact.

We know that the point of contact of a tangent line of a circle is that point of the line which is nearest to the centre. In other words the segment joining the centre with the point of contact is the shortest segment from the centre to the line and is therefore perpendicular to the tangent line.

This property gives us a method of constructing a tangent line having a given point of the circle as a point of contact.

All that we have to do is to draw a line through the given point perpendicular to the radius at the given point.

Since one and only one perpendicular can be drawn to a given line at a point on it we see that one and only one tangent line can be drawn passing through a given point on a circle.

Exercise 2.4

1. Frame true sentences by simply combining suitable words given in three groups A, B, C.

A	B	C
No point of a One point of a Two points of a Infinite points of a	tangent lie(s) secant lie(s)	on the circle inside the circle outside the circle

e.g. No point of a tangent to a circle lies inside the circle.

2. P is the centre of a cirlce of radius 5. Match the following:

The line L whose distance from P is 6	is a tangent.
The line M whose distance from P is 5	is a secant.
The line N whose distance from P is 4	is neither a secant nor a tangent to the circle.

3. State whether the following statements are true or false. Correct the statements in case they are false.
   (*a*) A line perpendicular to a radius of a circle is a tangent to the circle.
   (*b*) A secant of a circle is coplanar with the circle.
   (*c*) All tangents of a circle are equidistant from its centre.

4. **Given**: circle with centre O,
chord AB || chord CD. M is the mid point of chord AB. N is midpoint of chord CD.
To prove: O lies on seg MN.
[**Hint**: Join MO. Produce seg MO to meet chord CD in P and show that P is not different from N.]

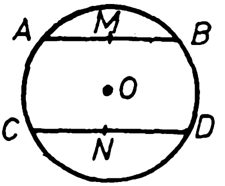


Fig. 2.41

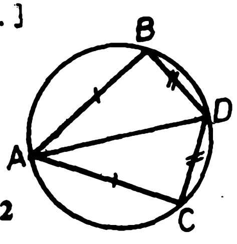


Fig. 2.42

5. **Given**: A circle with diameter AD.
$l(AD) = 10$, $m\angle BAD = 30^\circ$
$l(AB) = l(AC)$, $l(BD) = l(CD)$
Find: (i) perimeter of □ABDC
(ii) Area of □ABDC

6. **Given**: A circle with centre P
chord MN ≅ chord ML
seg PQ ⊥ chord MN
seg PR ⊥ chord ML
To prove: seg QN ≅ seg RL.

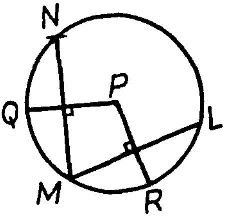


Fig. 2.43

7. Show that midpoints of congruent chords of a circle lie on another circle.

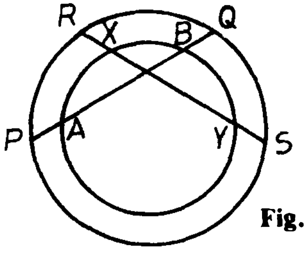


Fig. 2.44

8. **Given**: two concentric circles as in fig. 2.44.
chord PQ ≅ chord RS.
To prove: chord AB ≅ chord XY.

9. **Given:** i) a circle with centre P and radius 6
ii) $\triangle PAB$, $l(AB) = 6$, $A(\triangle PAB) = 15$
show : line AB is secant of the circle.

10. **Given:** i) a circle with centre P and radius o.
ii) $\triangle PCD$, $l(CD) = 6$, $A(\triangle PCD) = 18$
Show : line CD is a tangent to the circle.

11. **Given:** i) a circle with centre P and radius 6
ii) $\triangle PCD$ is a 30-60-90 triangle.
iii) $A(\triangle PCD) = 32\sqrt{3}$.
show : line CD is neither a secant nor a tangent line of the circle.

§ 2.12 Two or more coplanar circles:

We have seen that one and only one circle passes through three given non-collinear points. As a consequence of this property two different circles can not intersect in more than two points.

Given two circles in the same plane we have the following situations as shown in figures below.

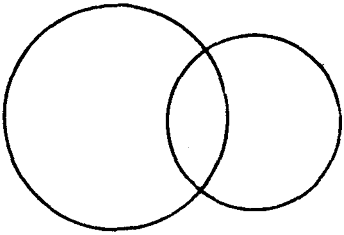


Fig. 2.45 (i)

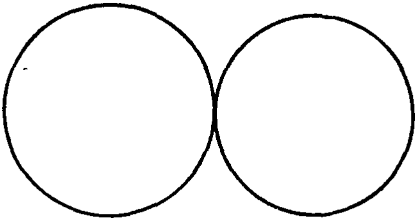


Fig. 2.45 (ii)*a*

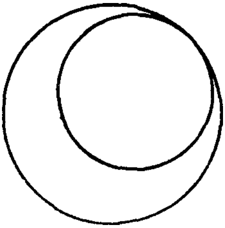


Fig. 2.45 (ii)*b*

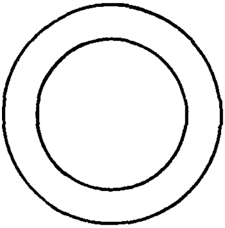


Fig. 2.45 (iii)*c*

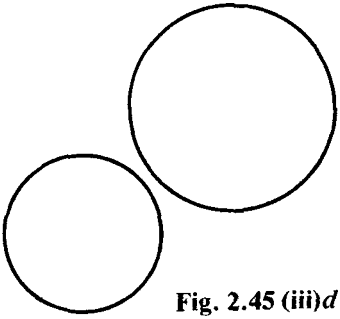


Fig. 2.45 (iii)*d*

(i) two circles intersect in two points.
(ii) two circles intersect in one point.
(iii) two circles do not intersect i.e. two circles do not have any point in common.

When two circles intersect each other in one and only one point we say that the two circles touch each other or that they are tangent to each other. We refer to such circles as tangent circles. When two circles touch each other their common point is called a point of contact.

Two non-coplanar circles may touch each other but when two coplanar circles touch each other they do so either internally or externally.

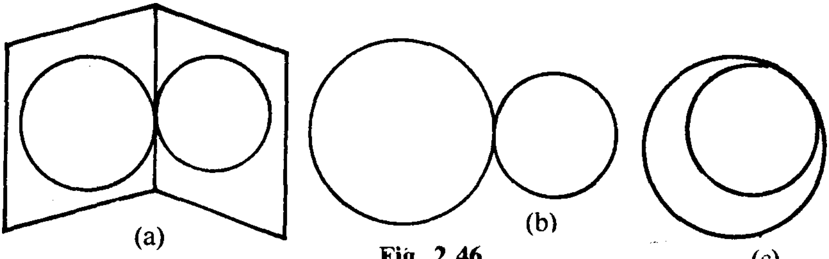


Fig. 2.46

If we observe two coplanar circles touching each other we see that there is always a line which is tangent to both the circles at their point of contact. On the other hand whenever there is line which is tangent to two circles at their common point, this point is the only point which is common to both and consequently the two circles are tangent to each other.

Thus two circles touch each other when and only when there is a line which is tangent to both the circles at a common point.

§ 2.13 Two circles intersecting in two distinct points:

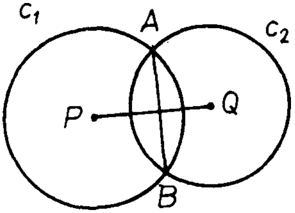


Fig. 2.47

In the adjoining fig (2.47) circle C_1 with centre P and circle C_2 with centre Q intersect in points A and B as shown.

Point P is equidistant from points A and B. Point Q is equidistant from points A and B.

So points P and Q lie on the perpendicular bisector of the common chord AB.

So we have,

If two coplanar circles intersect in two distinct points the line joining their centres is the perpendicular bisector of the segment joining the points of intersection.

§ 2.14 Two circles touching each other (internally or externally):

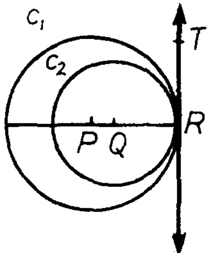


Fig. 2.48(*a*)

In the fig. 2.48 (*a*) & (*b*) two circles, circle C_1 with centre P and circle C_2 with centre Q are shown. Line T is the tangent line to both the circles at R, the point of contact. Line T is ⊥ to the radius of C_1 at R and line T is perpendicular to the radius of C_2 at R. But one and only one line can be drawn perpendicular to T at R.

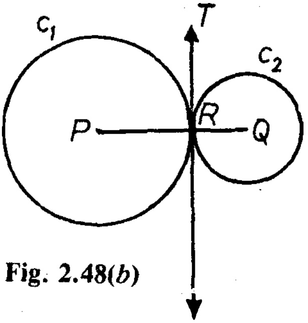


Fig. 2.48(*b*)

Thus,

When two circles touch each other, their centres and the point of contact lie on a line perpendicular to the common tangent at the point of contact.

§ 2.15 Tangents to a circle from a point outside:

In fig. 2.49, C is a circle with centre P. Q is a point outside C. seg RQ and seg SQ are tangent segments to the circle at the points R and S respectively. Since radius PR ⊥ line QR at R and radius PS ⊥ line QS at S, seg PQ subtends right angles at the points R and S, these points lie on a circle with seg PQ as diameter.

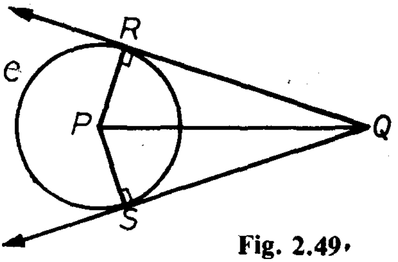


Fig. 2.49

In right angled triangles, △ PRQ and △PSQ

seg PR ≅ seg PS

seg PQ ≅ seg PQ

∴ △PRQ ≅ △PSQ (Hypo — side theorem)

∴ seg QR ≅ seg QS (corresponding sides of congruent triangles)

Thus we have,

From an external point of a circle two tangent segments to the circle can be drawn. These tangent segments are congruent and therefore have equal lengths.

In the above diagram show further that line PQ bisects ∠ RQS.

Exercise 2.5

1. Given a circle with centre P and radius 7. How many tangents to the circle can be drawn,
   (i) through Q if d(P,Q) = 6 ? (why ?)
   (ii) through R if d(P,R) = 7 ? (why ?)
   (iii) through S if d(P,S) = 8 ? (why ?)
2. Given two circles with centres P and Q and radii 8 and 5 respectively. In how many points will they intersect if the distance between their centres is—
   (i) equal to the sum of their radii, 8 + 5 = 13.
   (ii) equal to the difference of their radii, 8 − 5 = 3.
   (iii) less than 13 but greater than 3. (iv) zero.
3. In fig. 2.50 seg AB and seg AC are tangent segments to the circle with centre P.

Prove : (i) $l(AB) = l(AC)$
(ii) ∠ BAP ≅ ∠ CAP.

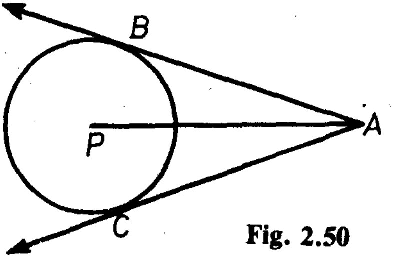


Fig. 2.50

4.

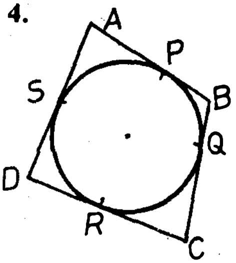


Fig. 2.51

In fig. 2.51, sides AB, BC, CD and DA of □ABCD touch a circle in P, Q, R, and S respectively.
Show that, $l(AD) + l(BC) = l(AB) + l(DC)$

5. In the figre 2.52 the sides of △ABC touch the circle with centre O and radius r at P,Q and R.
show that,
   (1) $l(AB) + l(CQ) = l(AC) + l(BQ)$
   (2) $A(\triangle OBC) = \frac{1}{2} \times l(BC) \times r$.
   (3) $A(\triangle ABC) = \frac{1}{2}$ (perimeter of △ABC) × r

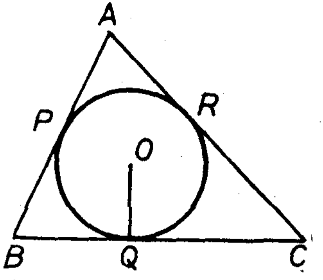


Fig. 2.52

6. In fig. 2.53 tangents at points M and N of a circle intersect at P.

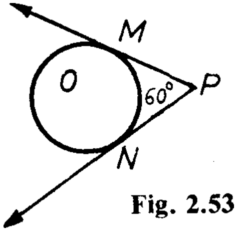


Fig. 2.53

$m\angle MPN = 60°$
$l(NM) = 10$. Find—
(i) radius of the circle
(ii) Area of □OMPN, if O is the centre.

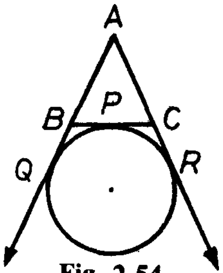


Fig. 2.54

7. In fig. 2.54, a circle touches the side BC of a △ ABC at P and side AB and side AC produced at Q and R respectively.
Prove that $l(AQ) = \frac{1}{2}$ (perimeter △ABC)

8. **Given** : 1. A circle with centre O and radius r
2. P is a point in the exterior of the circle.
3. $l(OP) = 2r$ and side PA, side PB are tangents to the circle at points A and B of it.

Prove : 1. △ABP is an equilateral triangle.
2. seg OP is perpendicular bisector of seg AB.

9.

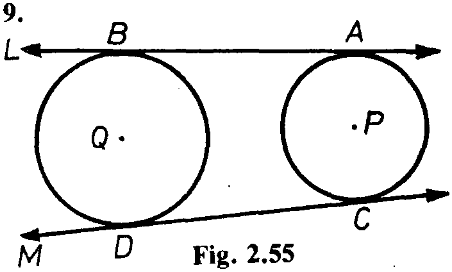


Fig. 2.55

Line L touches the circles with centres P and Q at the points A and B respectively and line M touches them at points C and D respectively.
Prove: $l(AB) = l(CD)$

10. In fig. 2.55, find A(□PABQ), if $l(AB) = 10$ and radii of the circles are 4 and 2 respectively.

11. **Given:** 1) Two circles with centres P and Q respectively touch at T.
2) Line L touches both the circles at A and B resepectively.
3) Tangent at T intersects line L at C.

Show: (1) C is the mid point of seg AB.
(2) seg BT ⊥ seg AT.

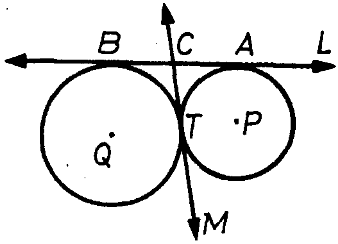


Fig. 2.56

12. In fig. 2.56, if $m \angle BTQ = 45^\circ$ show that the two circles are congruent.

13. In fig. 2.56, $m \angle BTQ = 45^\circ$ and if radii of both the circles are 5 each, find $A(\square QPAB)$.

14. **Given:**
(1) Two circles with centres P and Q respectively.
(2) Lines ABC and ADE are tangents to them at points B, C and points D, E respectively.

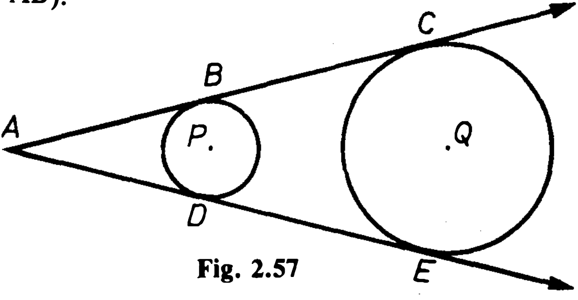


Fig. 2.57

Show: Points A, P and Q are collinear.

15. There are two circles, one with centre P and radius 4 and another with centre Q and radius 8. Lines AB and CD are tangents to both the circles intersecting at E, points A,B,C,D having points of contact as in fig. 2.58. If $l(PE) = 5$, $l(EQ) = 10$. Find: (a) $l(AB)$; (b) $l(CD)$.

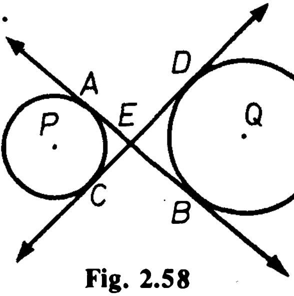


Fig. 2.58

16. Three circles with centres P, Q and R touch externally. If $l(PQ) = 5$, $l(QR) = 7$ and $l(PR) = 6$. Find the radius of each circle.

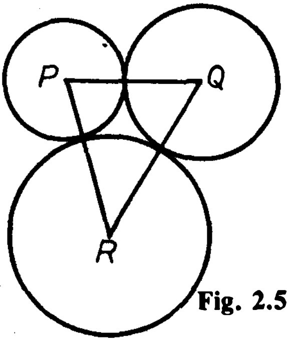


Fig. 2.59

17. If all the sides of a parallelogram touch a circle then show that the parallelogram is a rhombus.

18. Of the two congruent circles of radii 5, each passes through the centre of the other and intersect at P and Q. If the centres are denoted by A and B, find the area of rhombus APBQ.

19. In a circle with centre P, seg AB is a diameter and seg AC is any other chord. A secant through P parallel to seg AC intersects the tangent at C in a point D. Prove that line DB is a tangent to the circle at B.

20. Two circles touch at T and the smaller circle with centre A passes through B, the centre of the larger circle. Prove that the smaller circle bisects any chord of the larger circle which passes through T.

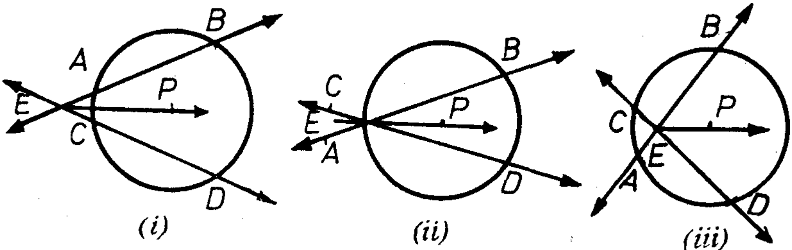
Miscellaneous Excercise - 2

1. Find the radius of the circle touching the sides of any equilateral triangle of perimeter 18. Also find the radius of the circumcircle of that triangle.

2. P is the centre of a circle. Lines AB and AC are tangents to it at B and C. If $m \angle BPC = 90^\circ$ and area of □PBAC is 25 then find the radius of the circle.

3. Three circles with equal radii touch each other. Show that the triangle formed by joining their centres is equilateral.

4. Chord AB of length 30 is at a distance of 8 from the centre of a circle. Find the distance from the centre of a chord CD of the same circle having length 16.



(i) (ii) (iii)

Fig. 2.60

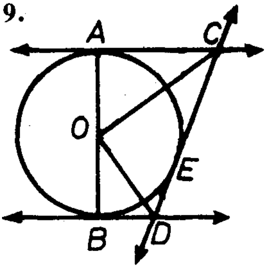
5. Secants AB and CD intersect at E. The chords intercepted by them are congruent. Show that ray EP bisects ∠ BED if P is the centre of the circle. [Fig. 2.60]

6. Point P is in the exterior of a circle with centre A. Line PT is a tangent to the circle and line PR is a secant of the circle. Show that ray PA does not bisect ∠ TPR.

7. Find the area of a quadrilateral MQPR formed by tangents PQ and PR of a circle with centre M and radius 8, if points Q and R are on the circle and $m\angle QPR = 60^\circ$

8. A chord AB of the larger of the two concentric circles is tangent to the smaller circle at the point P. Show that P is the midpoint of chord AB.

9.



Given: (1) A circle with centre O
(2) seg AB is a diameter of the circle.
(3) The line CD is a tangent to the circle intersecting tangents at A and B in the points C and D.

Prove: (i) $m \angle COD = 90°$
(ii) line AC || line BD.

Fig. 2.61

10. **Given:** (1) A circle with centre O.
(2) seg AB is a diameter of the circle.
(3) Line CD is a tangent to the circle intersecting perpendiculars to it from A at C and from B at D.

Prove: seg OC ≅ seg OD.

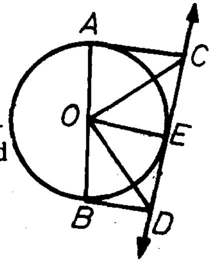


Fig. 2.62

□ □ □

Chapter 3

CIRCLE : ARC AND AREA

§ 3.1 Arc of a circle:

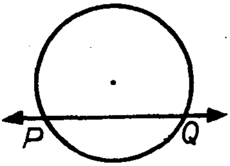


Fig. 3.1

Let P and Q be any two points on a circle C. The line joining these points divides the circle in two parts. Each part together with end points P and Q is called **an arc** of the circle. P and Q are said to be the end-points of the arcs

To distinguish between the two arcs, we take an arbitrary point on each arc as shown in the figure. We then name the arcs as arc PAQ and arc PBQ.

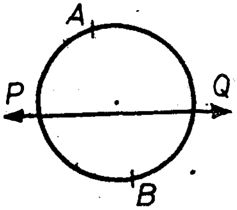


Fig. 3.2

When P and Q are the end points of a diameter of a circle, each arc, is called a **semi-circle**. In this case the arcs are congruent. An arc shorter than a semi-circle is called a **minor arc**. An arc larger than a semi-circle is called a **majore arc**. In fig. 3.4 arc PAQ is a major arc and arc PBQ is a minor arc.

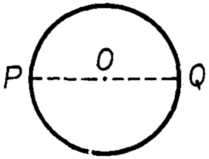


Fig. 3.3

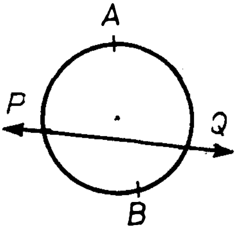


Fig. 3.4

§ **3.2 Measure of an arc:** Let P and Q be the endpoints of a minor arc PAQ of a circle C with centre O. $\angle$ POQ is called a **central angle**, subtended at the centre O by the arc PAQ.

$m\angle$ POQ is called the measure of minor arc PAQ. Correspondingly to the minor arc PAQ. .we have a major arc of the circle (say arc PBQ). Measure of a semi-circle is taken to be 180° and that of a circle is taken to be 360°. Then the measure of the major arc PBQ = 360° — measure of the minor arc PAQ. Measure of arc PAQ is written as *m*(arc PAQ).

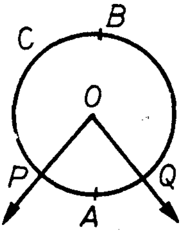
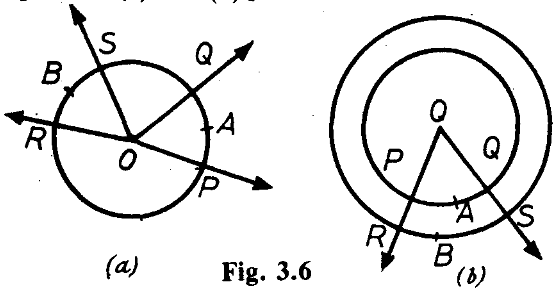


Fig. 3.5

In fig. 3.5 $m(\text{arc PAQ}) = m\angle POQ$.
and $m(\text{arc PBQ}) = 360° - m\angle POQ$.

Observe that - (1) Two different arcs of the same circle may have the same measure. (2) Two arcs of two different circles may have the same measure. [fig. 3.6(a) and (b)]



(a) **Fig. 3.6** (b)

$m(\text{arc PQ}) = m(\text{arc RBS})$ because $m\angle PQQ = m\angle ROS$.

§ **3.3** Intuitively speaking when two segments do not overlap and are not disjoint then these two segments can be added in the sense that a larger segment is obtained by adjoining the two given segments and the length of the new segment is the sum of the lengths of the given segments.

Similar consideration arises in respect of arcs of the same circle. We intuitively feel that when two arcs of the same circle are not disjoint but do not overlap these arcs can be adjoined to get a larger arc and the measure of the new arc is the sum of the measures of the given two arcs. We, therefore, have the following theorem.

Theorem: The arc addition theorem.

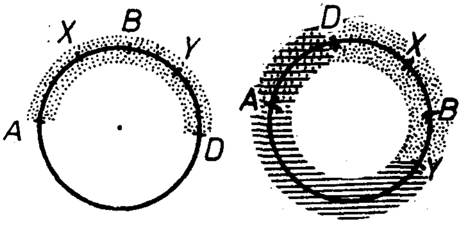


Fig. 3.7

If arc AXB and arc BYD are the arcs of the same circle having the end-point B as one and only one point in common, then

m(arc AXB) + m(arc BYD)
= m(arc ABD)

Note: The theorem states that the arcs have no point other than B in common i.e., we do not want a situation like arc AXB and arc BYD as in fig. 3.7 (b). Observe that if two arcs of the same circle have only their end points in common, from them we get the entire circle.

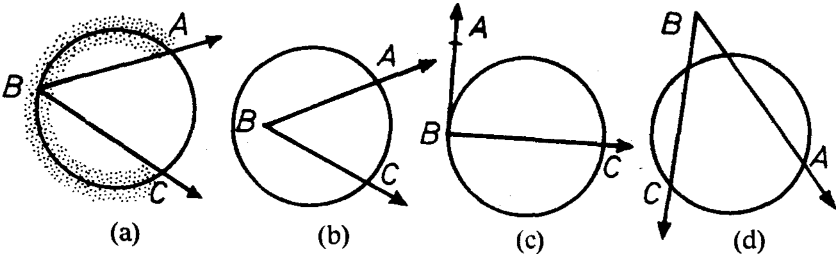
§ 3.4 Inscribed angle:

Given an arc of a circle an angle is said to be inscribed in the arc if

(i) each side of the angle contains one end point of the arc, and

(ii) the vertex of the angle is a point of the same arc other than its end-points.

Of the following figures, only in case (a) we can say that ∠ ABC is inscribed in an arc of the circle.



(a) (b) (c) (d)

Fig. 3.8

§ 3.5 Intercepted arc: An angle is said to intercept an arc, or an arc is said to be intercepted by an angle if (i) each side of the angle contains one of the end-points of the arc, and

(ii) all other points of the arc, are in the interior of the angle.

In the following figures, the arcs shown by shaded region are the arcs intercepted by ∠ ABC.

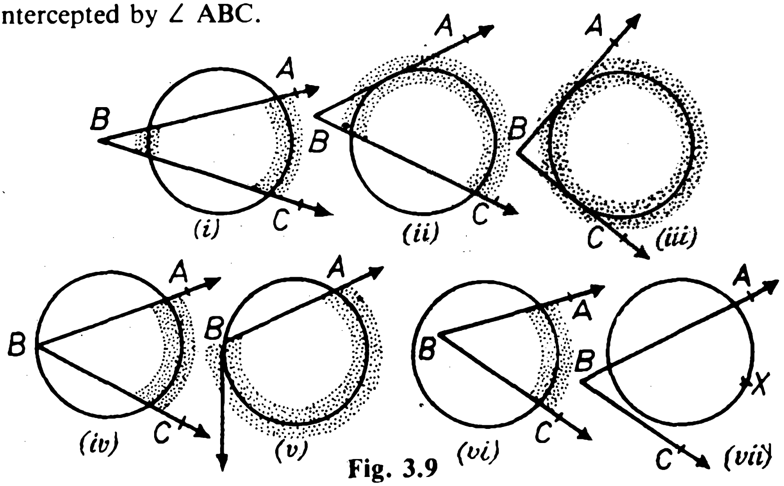


Fig. 3.9

In (i), (ii) and (iii) of fig. 3.9 ∠ ABC intercepts two arcs of the given circle.

In (iv), (v) and (vi) of fig. 3.9 ∠ ABC intercepts one arc on the given circle.

In fig. 3.9 (vii) we will not say that the ∠ ABC intercepts an arc AXB. From the above figures the following points are clear.

(*a*) An angle associated with the intercepted arc need not be an angle inscribed in an arc of the same circle.

(*b*) An angle inscribed in an arc of a circle necessarily intercepts an arc on the same circle.

(*c*) An angle can intercept two arcs on a circle.

There are some interesting relations between measures of intercepted arcs and measure of the angle which intercepts them, which are as follows.

1. If the vertex of such an ∠ ABC is in the exterior of the circle then *m* ∠ ABC = half the difference of the measures of the arcs intercepted by ∠ ABC. [Fig. (i), (ii), (iii)
2. If the vertex of such an ∠ ABC is on the circle then *m* ∠ ABC = half the measure of the arc intercepted by ∠ ABC. [Fig. (iv) & (v)]
3. If the vertex of such an ∠ ABC lies inside the circle then, *m* ∠ ABC = half the sum of the measures of arcs intercepted by ∠ ABC and by its vertically opposite angle ∠ DBE, as shown in fig. 3.10.

The proofs of all the above statements are based on the following two theorems. (§ 3.6 and § 3.10)

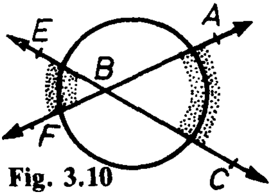
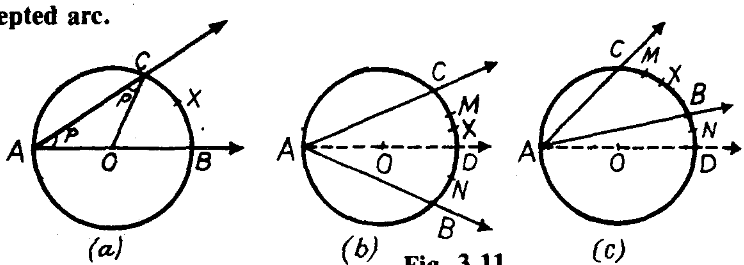


Fig. 3.10

§ 3.6 Theorem: Inscribed angle theorem:

The measure of an inscribed angle is half the measure of the intercepted arc.



(a) (b) (c)

Fig. 3.11

Given: (1) A circle with centre O.

(2) $\angle A$ inscribed in an arc BAC.

(3) Arc BXC intercepted by $\angle A$.

To prove: $m \angle BAC = \frac{1}{2} m$ (arc BXC)

Proff: There are three possible cases as shown in the figure 3.11.

Case 1: When the centre O is on $\angle BAC$ Join O and C.

$\triangle OAC$ is an isosceles triangle, Let $m \angle OAC = m \angle OCA = p$

Then by the property of an exterior angle of a triangle.

$m \angle BOC = p + p = 2p = 2\, m \angle BAC.$

$\therefore\ m \angle BAC = \frac{1}{2} m \angle BOC$

But $m \angle BOC = m$ (arc BXC)

$\therefore\ m \angle BAC = \frac{1}{2} m$ (arc BXC)

Case 2: When the centre O is in the interior of $\angle BAC$.

Let ray AO intersect the circle in the point D. Let us name the arc intercepted by $\angle CAD$ as arc CMD by taking a point on the arc. Similarly, let us name the arc intercepted by $\angle DAB$ as arc CND.

Therefore, using the result of case (I)

$m \angle CAD = \frac{1}{2} m$(arc CMD) .... (i)

$m \angle DAB = \frac{1}{2} m$(arc DNB) ....(ii)

Adding (i) and (ii)

$m \angle CAD + m \angle DAB = \frac{1}{2} m$(arc CMD) $+ \frac{1}{2} m$(arc DNB)

$\therefore\ m \angle BAC = \frac{1}{2}$ [m (arc CMD) + m(arc DNB)]

$= \frac{1}{2} m$(arc BXC)

Case 3: When centre O is exterior to $\angle$ BAC

As in case 2 let ray AO intersect the cirlce in the point D and let M and N be the points as in case 2. Using the result of case 1

$m\angle CAD = \frac{1}{2}\, m(\text{arc CMD})$ .... (i)

$m\angle DAB = \frac{1}{2}\, m(\text{arc DNB})$ .... (ii)

subtracting (ii) from (i)

$m\angle CAD - m\angle DAB = \frac{1}{2}\,[\, m(\text{arc CMD}) - m(\text{arc DNB})\,]$

$\therefore\ m\angle BAC = \frac{1}{2}\, m(\text{arc BXC})$

§ 3.7 The following corollaries of the inscribed angle theorem are useful:

Cor. 1: The angle inscribed in a semicircle is a right angle.

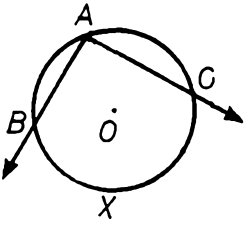


Fig. 3.12

Let $\angle$ CAB be inscribed in the semicircle CAB. Then it intercepts semicircle CXB.

$\therefore\ m\angle CAB = \frac{1}{2}\, m(\text{arc CXB})$

$\therefore\ m\angle CAB = \frac{1}{2} \times 180^\circ = 90^\circ$

Cor. 2: Angles inscribed in the same arc are congruent. Let $\angle$ CAB and $\angle$ CDB be inscribed in the arc CAB. Both these angles intercept the same arc CXB.

$\therefore\ m\angle CAB = \frac{1}{2}\, m(\text{arc CXB})$ .... (i)

$m\angle CDB = \frac{1}{2}\, m(\text{arc CXB})$ ....(ii)

from (i) and (ii) $m\angle CAB = m\angle CDB$.

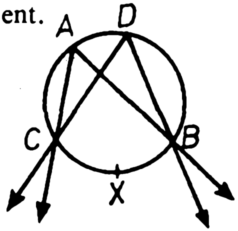


Fig. 3.13

§ 3.8 Cyclic Quadrilateral:

A quadrilateral is a cyclic quadrilateral if all its vertices lie on a circle.

Theorem: The opposite angles of a cyclic quadrilateral are supplementary.

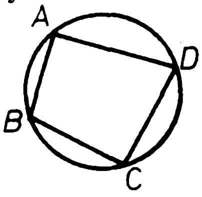


Fig. 3.14

Let ABCD be a cyclic quadrilateral.

To prove: $m\angle A + m\angle C = 180°$

$m\angle B + m\angle D = 180°$

Proof: Inscribed $\angle$ BAD intercepts arc BCD.

$\therefore m\angle BAD = \frac{1}{2} m(\text{arc BCD})$ .... (i)

Also, inscribed $\angle$ BCD intercepts are BAD

$\therefore m\angle BCD = \frac{1}{2} m(\text{arc BAD})$ ....(ii)

from (i) and (ii)

$m\angle BAD + m\angle BCD = \frac{1}{2}\,[\,m(\text{arc BCD}) + m(\text{arc BAD})\,]$

$\therefore m\angle BAD + m\angle BCD = \frac{1}{2} \times 360°$

$\therefore m\angle BAD + m\angle BCD = 180°$

or $m\angle A + m\angle C = 180°$

Similarly, we have $m\angle B + m\angle D = 180°$

§ 3.9 Some Solved Problems:

1. If a line segment joining two points subtends congruent angles at other two points lying on the same side of the line containing the segment, the four points lie on the same circle.

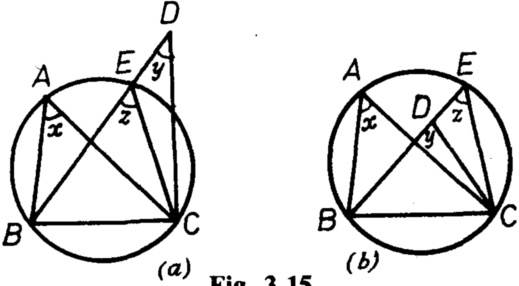


Fig. 3.15

Given: seg BC. Two points A and D on the same side of the line BC such that $m\angle BAC = m\angle BDC$

To prove: Points A, B, C and D lie on the same circle.

Proof: Let us assume that the four points do not lie on a circle.

For convenience let us say that the circle through points A, B and C, does not pass through D, and further that the circle meets seg BD (or seg BD produced) at E.

Also, let $m\angle BAC = x$, $m\angle BDC = y$, $m\angle BEC = z$.

Now $x = z$ (by inscribed angle theorem)

$x = y$ (Given)

$\therefore y = z$

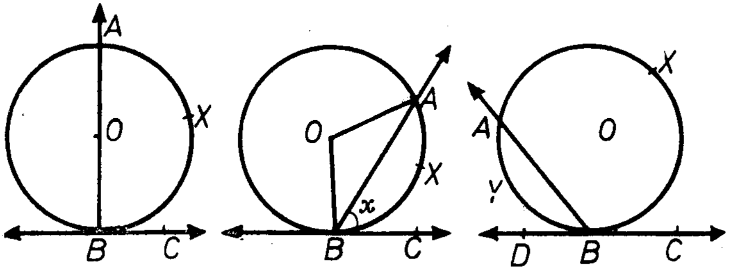
But this is impossible, since the exterior angle of a triangle in always greater than any of the remote interior angles.

Hence the circle passing through pts. A, B, and C must also pass through the pt. D. In other words the points A, B, C and D lie on the same circle.

Similarly the following theorem can be proved by the indirect method. **'If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.'**

§ 3.10 Tangent - Secant Theorem:

Given an angle with its vertex on the circle, if one side of the angle touches the circle and the other intersects the circle in two points, then the measure of the angle is half the measure of the intercepted arc.



(a) (b) **Fig. 3.16** (c)

Given: (1) A circle with centre O.

(2) $\angle$ ABC vertex B on the circle such that line BC touches the circle at B and the line BA intersects the circle again at A.

To prove: $m \angle ABC = \frac{1}{2} m(\text{arc } BXA)$

Proof: We shall consider three cases. $\angle$ ABC intercepts on the circle (*a*) a semicircle (*b*) a minor arc (*c*) a major arc.

In case (*a*). $m \angle ABC = 90°$ (why ?)

$m(\text{arc } AXB) = 180°$ (why ?)

$\therefore m \angle ABC = \frac{1}{2} m(\text{arc } AXB)$

In case (*b*), let $m \angle ABC = x$

Join O to B and O to A.

$m \angle OBC = 90°$ and

$m \angle ABC = x°$.

$\therefore m \angle OBA = m \angle OAB = (90 - x)°$

But in $\triangle$OAB,

$m \angle BOA + m \angle OBA + m \angle OAB = 180$

$\therefore m \angle BOA + (90 - x)° + (90 - x)° = 180$

$\therefore m \angle BOA = 180 - 180 + 2x = 2x$

But $m \angle BOA = m(\text{arc } AXB) = 2x = 2\, m \angle ABC$

$\therefore m(\text{arc } AXB) = 2\, m \angle ABC$ $\therefore m \angle ABC = \frac{1}{2} m(\text{arc } AXB)$

In case (*c*), take point D such that D – B – C.

Let $m \angle DBA = x$ and arc BYA is a minor arc (why ?)

$\therefore$ m(arc BYA) = $2x$

$\because$ $\angle$ DBA and $\angle$ ABC form a linear pair,

$\therefore$ $m \angle ABC = (180 - x)^\circ$

Also, as arc(BXA) is the larger arc opposite to minor arc(BYA),

$$\therefore m\,(\text{arc BXA}) = 360^\circ - 2x^\circ = 2(180^\circ - x)^\circ$$
$$= 2\, m \angle ABC$$
$$\therefore m \angle ABC = \tfrac{1}{2}\, m(\text{arc BXA})$$

Thus we have proved the theorem in all cases.

Exercise 3.1

1. Prove in each of the following figures $m \angle ABC = \frac{1}{2}$ [Difference of the measures of arcs intercepted by $\angle$ ABC]

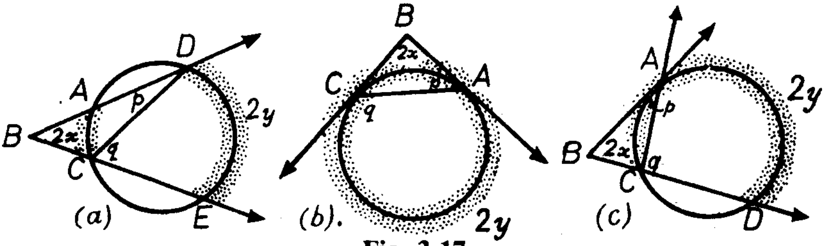


Fig. 3.17

Hint: Let measures of the intercepted arcs be $2x$ and $2y$ as shown in the figures.

In each figure,

$p = x$ and $q = y$ (why ?)

by exterior angle theorem in each case,

$$m \angle B = q - p = y - x$$
$$= \tfrac{1}{2}(2y - 2x)$$
$$= \tfrac{1}{2}\ [\text{ Difference of the measures of the intercepted arcs }]$$

2. Prove in each of the following cases $m \angle ABC = \frac{1}{2}$ measure of the intercepted arc.

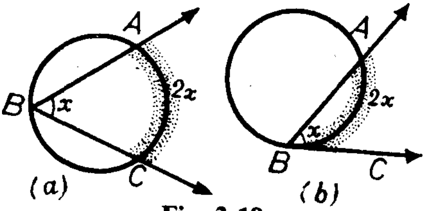


Fig. 3.18

Hint: We have already proved both these cases as theorems. Give the proof in your own language.

3. Prove in the fig., $m \angle ABC = \frac{1}{2}$ [the sum of the measures of arcs intercepted by $\angle ABC$ and $\angle DBE$]

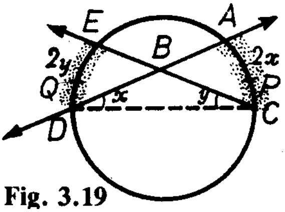


Fig. 3.19

Hint: $m \angle ECD = y$;

$m \angle ADC = x$ (why ?)

$\therefore m(\text{Ext. } \angle ABC) = x + y$

$= \frac{1}{2}(2x + 2y)$

$= \frac{1}{2}$ [m(arc APC) + m(arc EQP)]

§ 3.11 Congruent arcs:

Two arcs of the same circle (or congruent circles) having equal measures are called congruent arcs.

This also means that,

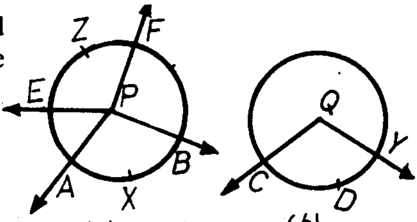
congruent arcs of the same circle (or congruent circles) subtend congruent angles at their centres.

The two circles with centres P and Q respectively in the figure are given to be congruent.

Also $m \angle APB = m \angle EPF$
$= m \angle CQY$.

Then,

arc AXB ≅ arc EZF ≅ arc CDY.



(a) **Fig. 3.20** (b)

However, in the following concentric circles with centre O,

$m \angle AOB = m \angle COD$, but arc (AMB) is not congruent to arc CND, obviously because the circles are not congruent.

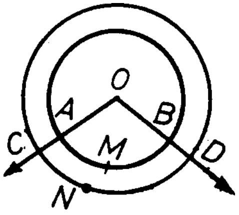


Fig. 3.21

In the following figures, the two circles are congruent and arc(AXB) ≅ arc(CYD)

- Can you prove,
  (i) $\angle AEB \cong \angle CFD$
  (ii) $\angle AXB \cong \angle CYD$

Let P and Q be the centres of the circles.

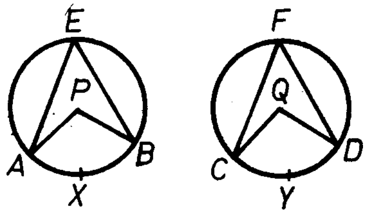


Fig. 3.22

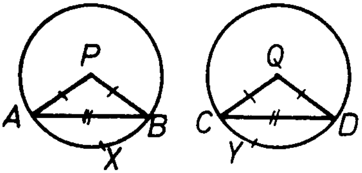
Let $m \angle \text{AMB} = 2x = m \angle \text{CND}$

$\therefore\ m \angle \text{AEB} = x = m \angle \text{CFD}$ (why?)

also $m \angle \text{AXB} = 180 - x$

$= m \angle \text{CYD}$ (why?)

§ 3.12 In the following figures, the two circles with centres P and Q are congruent and chord AB ≅ chord CD. Is arc(AXB) ≅ arc(CYD) ?



(a) (b)

In other words

Theorem: If two chords of congruent circles (or of the same circle) are congruent, then the corresponding arcs are congruent.

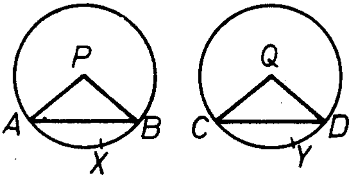
Proof: △APB ≅ △CQD (S.S.S.)

$\therefore$ ∠ APB ≅ ∠ CQD (corresponding angles of congruent triangles.)

$\therefore$ arc(AXB) ≅ arc(CYD) (why?)

Theorem: (Converse of theorem in § 3.12)

If two arcs of congruent circles (or in the same circle) are congruent, the corresponding chords are congruent.



(a) **Fig. 3.24** (b)

Given: (i) Two congruent circles with centres P and Q.
(ii) arc(AXB) ≅ arc(CYD) as shown in the fig.

To prove: chord AB ≅ chord CD.

Proof: arc(AXB) ≅ arc (CYD) ....Given

$\therefore$ ∠ APB ≅ ∠ CQD [Central angles of congruent arcs of congruent circles]

seg AP ≅ seg CQ ....(radii of congruent circles)

seg BP ≅ seg DQ ....(radii of congruent circles)

△APB ≅ △CQD ....By S.A.S.

seg AB ≅ seg CD [corresponding sides of congruent triangles]

Excercise 3.2

1.

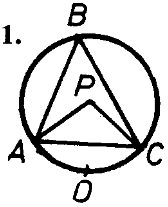


Fig. 3.25

Given: (1) P is the centre of a circle.
(2) $m\angle \text{ABC} = 50°$

Find: 1) The measures of the angles of △APC.
2) m(arc ADC)
3) m(arc ABC)

2.

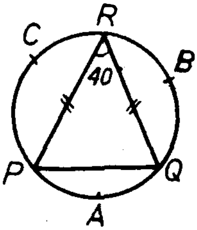


Fig. 3.26

Given: (1) chord PR ≅ chord RQ
(2) $m\angle PRQ = 40^\circ$

Find: (1) m(arc PAQ)
(2) m(arc RBQ)
(3) m(arc RPQ)

3. In fig. 3.27 chord AB ≅ chord CD then arc APC ≅ arc(DQB)

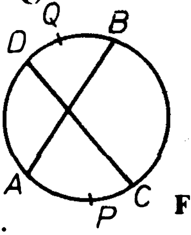


Fig. 3.27

4.

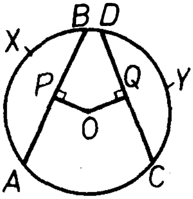


Fig. 3.28

Given: (1) O is the centre.
(2) seg OP ⊥ chord AB.
(3) seg OQ ⊥ chord CD.

If (i) l(OP) = l(OQ) then show that m(arc AXB) = m(arc CYD) and m(arc AYB) = m(arc CXD)

If (ii) l(OP) > l(OQ) then show that m(arc AXB) < m(arc CYD) and if (iii) l(OP) < l(OQ) then show that m(arc AXB) > m(arc CYD)

5. **Given:** chord BD ≅ chord CE.
$m\angle CBA = 50^\circ$.

Find: $m\angle BCA$, $m\angle BDE$ and show seg BC ∥ seg DE.

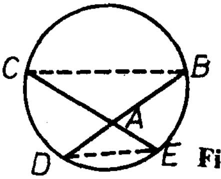


Fig. 3.29

6.

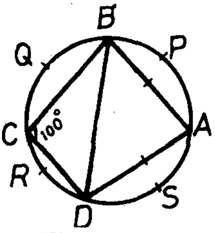


Fig. 3.30

Given: (1) ABCD is a cyclic quadrilateral.
(2) l(AB) = l(AD)
(3) $m\angle BCD = 100^\circ$

Find: (1) m(arc BAD)
(2) m(arc BCD)
(3) m(arc BPA)
(4) An arc congruent to arc BPA.

7. **Given:** □ ABCD is a cycle trapezium.
seg AB ∥ seg CD and
$m\angle ABC = 80^\circ$

Find: (1) m(arc ABC)
(2) $m\angle BAD$

and show that (3) arc BXC ≅ arc AYD.

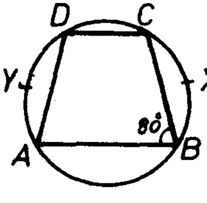


Fig. 3.31

8. **Given** - (1) cyclic □ABCD

(2) chord AB ≅ chord CB
chord DA ≅ chord DC

(3) $m\angle ADC = 3x$, $m\angle ABC = 2x$

Calculate the measure of each angle of □ABCD.

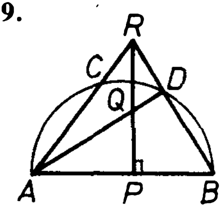
9. 

Fig. 3.32

Given: (1) arc ACDB is a semicircle on diameter AB

(2) ray AC and ray BD intersects at R.

(3) perpendicular RP on seg AB intersects seg AD at Q.

To prove: (1) □QDBP is cyclic

(2) □RCQD is cyclic.

[**Hint:** Use, perpendiculars from the vertices of a triangle meet in a point.]

10. **Given:** (1) a circle with centre P

(2) tangent T at a point Q on the circle.

(3) $m\angle RQS = 30°$

Find: (1) m(arc RAQ)

(2) $m\angle RAQ$

(3) $m\angle RPQ$.

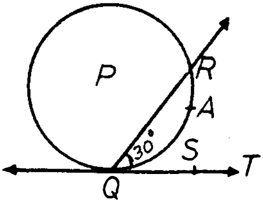


Fig. 3.33

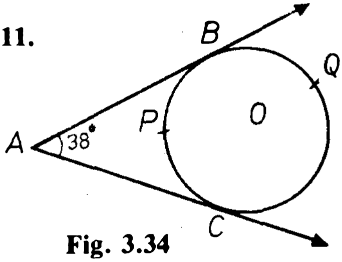
11. 

Fig. 3.34

Given: 1) AB and AC are tangents to the circle at points B and C of it.

2) $m\angle BAC = 38°$

Find: (i) m(arc BPC)

(ii) m(arc BQC)

(iii) $m\angle OBC$.

12. **Given:** (i) m(arc BYD) = 170°

(ii) m(arc PRQ) = 65°

Find: m(arc BXC)

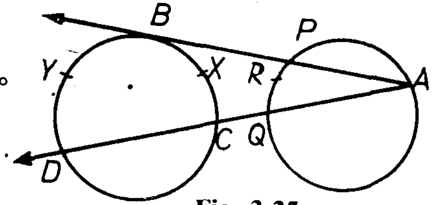


Fig. 3.35

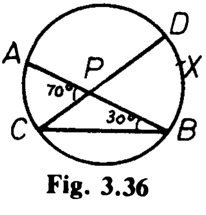
13. 

Fig. 3.36

Given: (1) $m\angle ABC = 30°$

(2) $m\angle APC = 70°$

Find: m(arc DXB)

14.

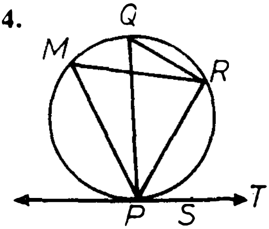


Fig. 3.36

Given: (1) T is a tangent at P

(2) seg PQ is a diameter.

(3) $m\angle$ PRS = 50°

Find: (1) $m\angle$ RQP

(2) $m\angle$ RMP

15. Given: 1) line PT is a tangent at P.

ray PS bisects $\angle$ RPT.

Show: arc RXS $\approxeq$ arc PYS.

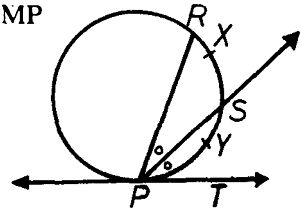


Fig. 3.38

16.

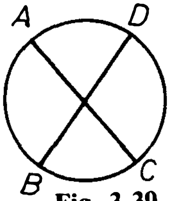


Fig. 3.39

Given: chords AC and BD bisect each other.

Prove: (1) seg AC and seg BD are diameters.

(2) □ ABCD is a rectangle.

(3) If dia. AC ⊥ dia.BD then □ ABCD is square.

17. Given: chord AB > chord CD

Prove: seg BE > seg DE
and seg CE < seg AE

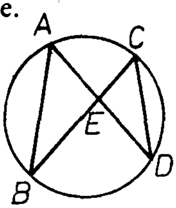


Fig. 3.40

18.

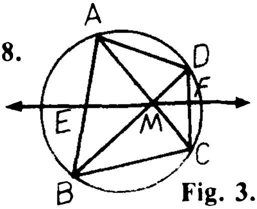


Fig. 3.41

Given: (1) In cyclic □ ABCD, diagonals AC and BD intersect at M at right angles.

(2) E is midpoint of seg AB.

To prove: Line EM ⊥ line DC.

19. Prove that the exterior angle formed by producing a side of a cyclic quadrilateral is congruent to the interior opposite angle.

20. If an exterior angle of a quadrilateral is congruent to its remote interior angle prove that the quadrilateral is cyclic.

21. Two circles intersect in A and B. Line PQ through A intersects the circles in P and Q and line RS through B intersects it at R and S as shown in fig. 3.42.
Prove: Line PR || line QS.

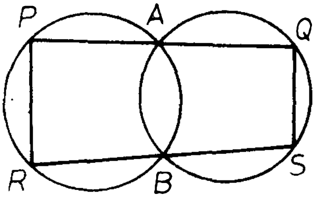


Fig. 3.42

22. Given : 1) seg AD $\parallel$ seg BC

2) seg AB $\cong$ seg DC

Prove : $\square$ABCD is cyclic.

23.

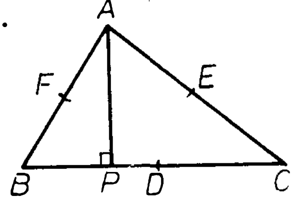


Fig. 3.43

Given: In $\triangle ABC$ points D, E and F are mid points respectively of sides BC, CA and AB.

seg AP $\perp$ side BC. B-P-C.

Prove: Points P, D, E and F are concylic.

24. Given : (1) $\triangle ABC$, $l(AB) = l(AC)$

(2) bisectors of $\angle B$ and $\angle C$.

meet seg AC in D and seg AB in E respectively.

Prove: B, C, D and E lie on a circle.

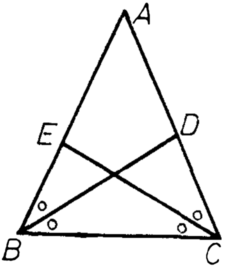


Fig. 3.44

25.

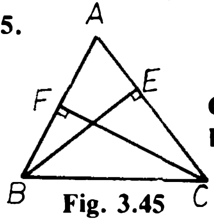


Fig. 3.45

Given : seg CF & seg BE are the altitudes in $\triangle ABC$

Prove : B,C,E and F lie on a circle.

§ 3.12 Segment and sector of a circle:

Let AB be a chord of a circle with centre O.

The chord AB divides the circular region in two parts, each of which is called a segment of the circular region or briefly a segment of the circle. A chord which is not a diameter divides a circular region into **minor segment** and **major segment**.

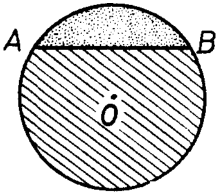


Fig. 3.46

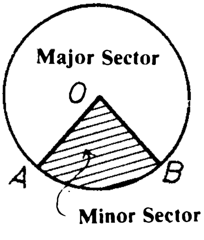


Fig. 3.47

Let seg OA and seg OB be two radii of a circle whose centre is O. The angle determined by these radii divides the circular region into two parts, each of which is called a sector of the circular region or briefly a sector of the circle. That sector which contains the minor arc is called a **minor sector** and the other one is called a **major sector**.

§ 3.13 Circumference of a circle:

A man completes a round about a circular garden. What distance has he walked ? A wooden wheel of a cart is to be fitted with a steel ring. What would be the length of the steel ring ?

Take a wooden wheel and roll it along a straight road without allowing it to slide. At the start mark the point on the road at which the wheel touches it and also the corresponding point on the wheel. Now roll the wheel so that the point marked on the wheel again comes in contact with the road. Mark the corresponding point on the road. The length of the line segment joining the two marked points on the road would be the circumference of the wheel. Find the ratio of the circumference with the diameter of the wheel. Repeat this experiment. This ratio comes out to be the same each time.

We say that,

"The circumference of a circle bears a constant ratio to its diameter."

The constant ratio is called 'pi' written as π. We write,

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi .$$

or circumference = πD, or circumference = $2 \pi R$
where D = diameter; R = radius.

§ 3.14 Through what angle do we rotate a circle; if it rolls through one third of its circumference. Naturally through 120 degrees. In that case, the measure of the arc so rolled is 120° and the length of the arc is one third of its circumference. Now, fill up the following table.

Part of the circumference of the circle rolled	Measure of the corresponding arc	length of the corresponding arc.
$\frac{1}{4}$ th	$\frac{360}{4} = 90^\circ$	$\frac{1}{4}$ th of the circumference
$\frac{1}{5}$ th	$\frac{360}{5} = 72^\circ$	$\frac{C}{5}$. c = circumference
......	$\frac{360}{\dots} = 60^\circ$	$\frac{C}{6}$
......	.. = 45°	......
......	.. = 30°	......

The table helps you to make a following guess:

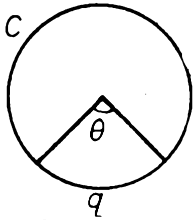


Fig. 3.48

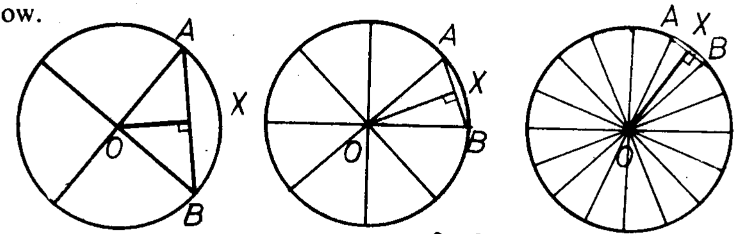
$$\frac{360}{\text{measure of an arc}} = \frac{\text{circumference}}{\text{length of the arc}}$$

If we denote circumference by C, measure of an arc by θ and length of arc by q then we get the formula $\frac{360}{\theta} = \frac{C}{q}$ or $q = \frac{C\theta}{360}$

$$C = \frac{360}{\theta} q$$

§ 3.15 Area of a circle:

We divide a circular region into 4, 8, 16, congurent sectors as shown below.



$l(\text{arc AB}) = \frac{2\pi R}{4}$ $l(\text{arc AB}) = \frac{2\pi R}{8}$ $l(\text{arc AB} = \frac{2\pi R}{16}$

Fig. 3.49

Every time compare the difference between the area occupied by the triangle OAB and the sector OAB. You realize that as the circular region is divided into more number of sectors the difference between the lengths of chord AB and the corresponding minor arc becomes less and less and consequently difference between areas of △OAB and sector OAB becomes less and less. If we divide the circular region into n congruent sectors and n becomes very very large, area of the sector OAB can be thought as almost being equal to the area of △OAB, with height r and base $\frac{2\pi R}{n}$

$$\therefore \ A(\triangle OAB) = \frac{1}{2} \times \frac{2\pi R}{n} \times R$$

$$\therefore \ A(\text{sector } OAB) = \frac{1}{2} \times \frac{2\pi R}{n} \times R = \frac{\pi R^2}{n} \ \ldots\ldots\ldots (1)$$

If we take all n congruent sectors, we get the area of the complete circular region,

Area of the circle $= n \times$ A(sector OAB)

$$= n \times \frac{\pi R^2}{n} = \pi R^2$$

$$\boxed{\text{Area of a circle of radius R is } \pi R^2}$$

Observe, step (1) again

A(sector OAB) $= \frac{1}{2} \times \frac{2\pi R}{n} \times R$

Here, $\frac{2\pi R}{n}$ is the lenght of the arc AB which we denote by q.

So we get.

$$\boxed{\text{A(sector OAB)} = \frac{1}{2} qR}$$

But, $q = \frac{C\theta}{360} = \frac{2\pi R\theta}{360}$ (where $C = 2\pi R$, $\theta = m(\text{arc AXB})$)

$\therefore$ A(sector OAB) $= \frac{1}{2} \times \frac{2\pi R\theta}{360} \times R$

$\therefore$ A(sector OAB) $= \pi R^2 \frac{\theta}{360}$

or $\frac{\text{A(sector OAB)}}{\text{Area of a circle}} = \frac{m(\text{arc AXB})}{360}$

Excercise 3.3

1. In the following table some information is given about circles. Fill up the table.

S.N.	Radius	Diameter	Circumference	Area
1	3.5			
2		$\frac{7}{11}$		
3			$\frac{22}{7}$	
4				1.54

2. In the following table information is given about an arc of a circle. Fill up the table.

S.N.	Radius	Length of an arc	Measure of an arc	Area of sector
1	5		30	
2	2.5		150	
3	4.2	$2.8\ \pi$		
4	12			$120\ \pi$.
5		$\frac{26}{5}\ \pi$	75	
6		$2\ \pi$		$5\ \pi$

3. Find the area of a circle touching all the three sides of an equilateral triangle of side 12 cm. Also find the area of its circumcircle.

4. The side of a square is 10 cm. Find the area of the circumscribed circle and also of the circle touching all its sides (inscribed circle.)

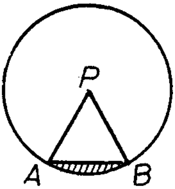
5. 

Fig. 3.50

Find the area of the segment of a circle with centre P, given that $m\ \angle\ APB = 60°$ and radius of the circle is 12.

6. The measure of an arc AXB of a circle of radius 12 cm is 45°. Find area of the segment.

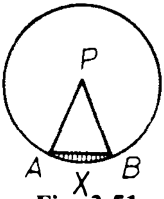


Fig. 3.51

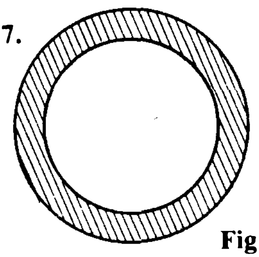
7. 

Fig. 3.52

The area of a circular road (as shown by shaded portion) is 22176 sq. metre. If the diameter of the outer circle is 280 m find the width of the road.

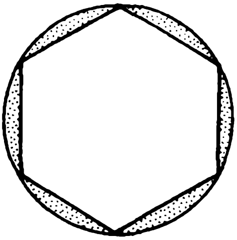


Fig. 3.53

8. A regular hexagon is inscribed in a circle of radius 10 cm. Find the area of the shaded portion.

9. The circumference of a circular playground is 528 m. Find the total expenditure of repairs of the ground at the rate of 50 paise per sq. meter.

10. A man can run with a speed of 15.84 km/hr. He completes 12 rounds of a circular ground in one hour. Find the area of the ground.

11. The length of the side of a square is 42 cm. Find the area of the portion between the square and its incircle

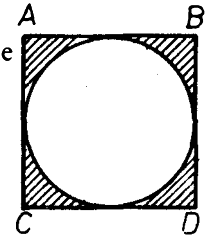


Fig. 3.54

12. The radii of two circles are 6 m and 8 m respectively. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

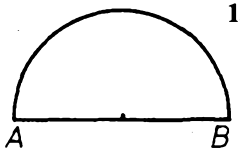


Fig. 3.55

13. A man by walking diametrically across a circular grass-plot takes 45 sec. less than if he had kept to the circular path round the outside. He walks 80 meteres a minute. What is the diameter of the grass-plot ?

14. The radius of the inner boundry of a plane circular ring is 14 cm. The area of the ring is 100 sq cm. Find the radius of the outer boundary.

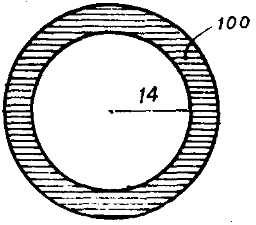


Fig. 3.56

15. In a circle of unit radius a regular hexagon is inscribed. In this hexagon is inscribed another circle. Find correct to four decimal places the area of the ring enclosed between the two circles.

[Take $\pi \approx 3.14159$]

16. In a circle of radius 65 m, two parallel chords are drawn on the same side of the centre and measuring 126 m and 112 m respectively. Find the distance between them.

17. Three congruent circles are so placed that each touches the other two. If the radius of each circle is 1 m find the area of the unoccupied space among them.

18. Two circular wheels of radii r_1 and r_2 and circumferences c_1 and c_2 respectively cover the same distance in the same interval of time. If the number of revolutions made by them in this time are n_1 and n_2 respectively, then show that

$$\frac{n_1}{n_2} = \frac{r_2}{r_1} = \frac{c_2}{c_1}$$

19. The diameter of a rear wheel of a horse-cart is 1.2 m. In a certain distance, the rear wheel makes 50 revolutions, while the front wheel makes 200 revolutions. What is the diameter of the front wheel ?

20. If the wheel of a bus with 90 cm diameter makes 210 revolutions per minute, find the speed of bus in km/hr.

Miscellaneous Exercise - 3

1. In the adjoining figure, arc DCB is an arc of a circle with centre A. $m \angle CBD = 45°$. Prove that seg DA is perpendicular to seg CA.

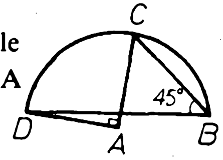


Fig. 3.57

2. Show that an angle inscribed in a major arc is acute and that which is inscribed in a minor arc is obtuse.

3. Prove that the circle drawn by taking the diameter as the hypotenuse of a right triangle passes through the opposite vertex.

4. A circle drawn on one side of a rhombus as diameter passes through the point of intersection of its diagonals.

5. Show that a cyclic parallelogram is a rectange.

6. Show that the quadrilateral formed by the angle bisectors of a quadrilateral is cyclic.

7. In the fig. 3.58 chord AB || chord CD. Show that, arc AXC ≅ arc BYD.

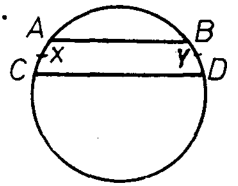


Fig. 3.58

8. In the fig. 3.59 diagonals AC and BD of the cyclic □ABCD are congruent.
Show that $l(AD) = l(BC)$ and seg AB ∥ seg DC
[Hint: If congruent chords of a circle intersect inside the circle corresponding parts are congruent.]

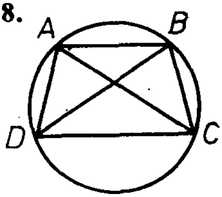


Fig. 3.59

9. In one pair of opposite sides of a cyclic quadrilateral is congruent then show that the diagonals are also congruent.

10. In fig. 3.60, seg AB is a chord of a circle. ∠ BAQ ≅ ∠ ACB.
[Point C is on the circle and is not on Q-side of AB.]
Show that seg AQ touches the circle at A.

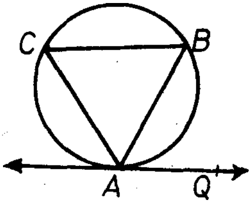


Fig. 3.60

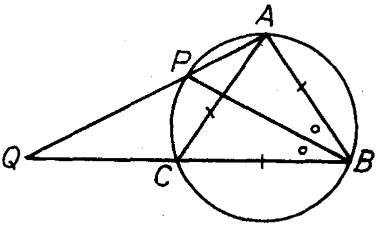


Fig. 3.61

11. **Given:**
(1) chord AB ≅ chord AC ≅ chord BC
(2) ∠ ABP ≅ ∠CBP

Prove:
seg CQ ≅ seg CA.

12. **Given:** Two non-congruent circles touch externally at T.

A segment AB passing through T intersects them in A and B respectively. Lines L and M are tangents to the circles at A and B respectively. Show that line L ∥ line M.
How are L and M related if the circles touch internally ?

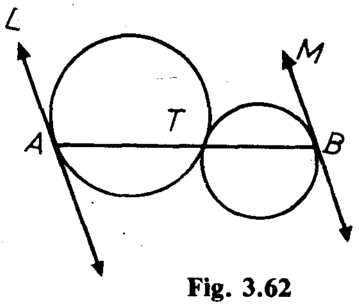


Fig. 3.62

13. Two circles intersect in points P and Q. A secant passing through P intersects the circles in A and B respectively. Tangents to the circles at A and B intersect at T. Prove that A, Q, T and B lie on a circle.

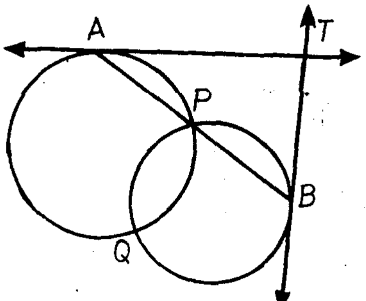


Fig. 3.63

14. Find the measure of that arc of a circle whose length is equal to its radius.
15. Find the length of the side of an equilateral triangle inscribed in a circle of radius 4.
16. A circular hole is to be cut in a circular disc of diameter $3\sqrt{3}$, so that the weight may be reduced by one third. Find the diameter of the hole.
17. The radius of a circle is 18. Find the radius of another circle of one fifth its area.
18. The radius of a circle is 20. It is required to draw three concentric circles in such a manner that the whole area may be divided into four parts of equal area. Find their radii.

□ □ □

Chapter 4

GEOMETRIC CONSTRUCTIONS

§ **4.1** The growth of the present scientific culture owes immensely to ancient Greek people. Greek mathematicians mainly engaged themselves in attacking geometrical problems. They had very little interest in arithmetic. They looked down on arithmetic as the field in which only merchants would be interested. Thus, inspite of having reached a very high stage of development in geometry its growth was stunted because they neglected to take interest in properties of numbers. It was only when D'escartes, the French mathematician, started using algebraic methods to study geometry, did this science start progressing again. It appears that because of their disrespect for numbers, Greeks had forbidden the use of a measuring scale, a protractor and a setsquare in construction of geometrical figures. Greeks insisted that in any geometrical construction use of only a ruler without numbers, compass and a divider was permissible. Even inspite of this restriction it is surprising that they could develop the science of geometrical constructions to a very great extent.

It is creditable that a large variety of constructions can be performed with limited instruments. Some of these constructions are listed below.

1. **To draw a circle of the given radius.**

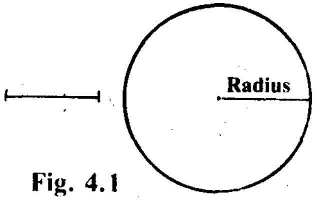


Fig. 4.1

2. **To draw a segment congruent to a given segment.**
   seg AB is a given segment.
   Draw, seg CD ≅ seg AB

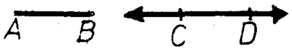


Fig. 4.2

seg AB ≅ seg CD OR
$l(AB) = l(CD)$

3. **To draw a segment twice, thrice, in length of the given segment.**
   Segment PQ is a given segment.
   Draw seg RS such that
   $l(RS) = 2l(PQ)$,
   $l(RT) = 3l(PQ)$.

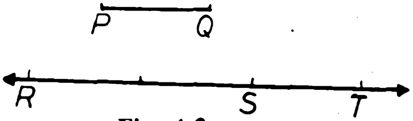


Fig. 4.3

$l(RS) = 2l(PQ)$
$l(RT) = 3l(PQ)$

4. To bisect a given segment.
Bisect the given segment AB.

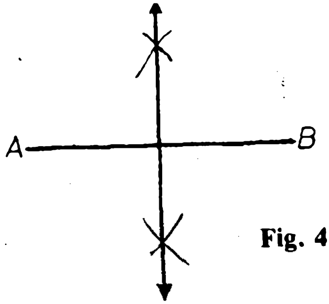


Fig. 4.4

5. To construct an angle congruent to the given angle.
Draw ∠ PQR congruent to ∠ BAC.

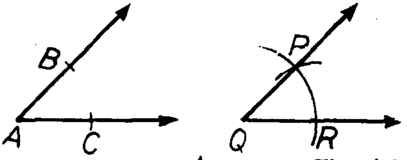


Fig. 4.5

6. To bisect the given angle.
Bisect ∠ ABC.

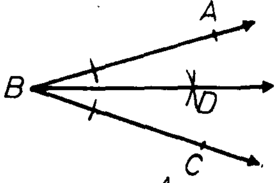


Fig. 4.6

7. To draw a line parallel to a given line from a point external to it.

Given line L and a point P out side it. Through point P, draw a line M parallel to line L, with the help of set square.

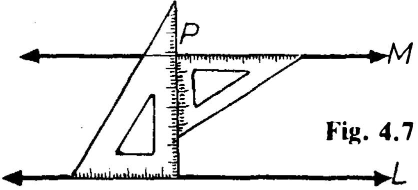


Fig. 4.7

8. To draw a line parallel to a given line from a given point by using alternate angle test, with the help of compass.

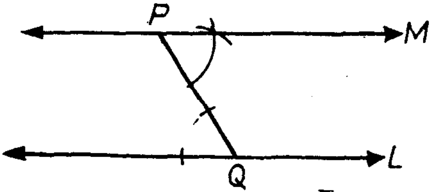


Fig. 4.8

9. To divide a given segment in congruent parts.

To divide the segment AB in 5 congruent parts.

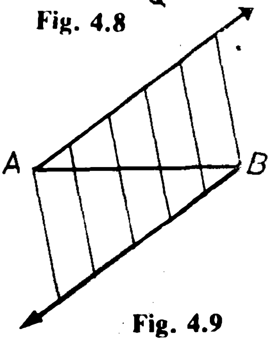


Fig. 4.9

10. To draw a perpendicular to a given line through a given point on it.

P is a point on the line L. Through P draw a perpendicular line to the line L.

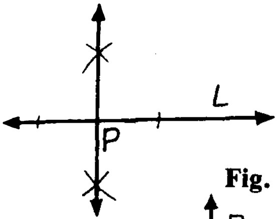


Fig. 4.10

11. To draw a perpendicular to a given line through a point lying outside it.

P is the point out side a line L. Through P draw a perpendicular line to L.

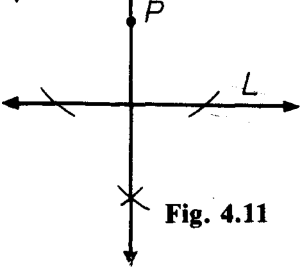


Fig. 4.11

12. To construct a right angled triangle when its hypotenuse and one side is given.

In ΔPQR,

$m\angle PRQ = 90°$, l(seg PR) and length of the hypotenuse is given. Then construct ΔPQR.

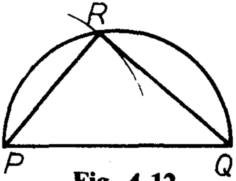
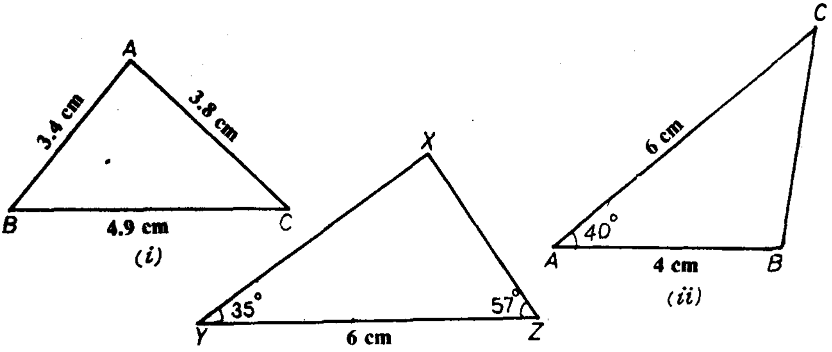


Fig. 4.12

Exercise 4.1

1. Given $\angle PQR$, construct $\angle XYZ$ congruent to it.
2. Construct an angle of measure 75° and bisect it.
3. Take a segment AB of length 5 cm Divide seg AB in 7 congruent parts.
4. Construct triangles, congruent to the triangles shown in the Figures below.



(i) (ii) (iii)

Fig. 4.13

5. Draw a segment AB of length 8.4 cm. Take a point P on segment Ab such that l(AP) = 2.6 cm. At the point P draw a perpendicular to the line AB.
6. Construct △ ABC such that l(AB) = 7.5 cm. l(BC) = 12 cm and l(CA) = 7.5 cm. Take the mid-point M of side AB. With centre M draw a circle of radius = l(MA) intersecting the side BC in point P. Find l(BP).
7. Construct △ PQR such that l(PQ) = 8.6 cm l(QR) = 6.4 cm, l(PR) = 8.6 cm. Draw seg PK ⊥ seg QR. (Point K on side QR). M is the mid-point of the side PQ of △ PQR. Draw a circle with centre 'M' and radius = l(PM). Find l(PM).
8. Draw an equilateral triangle XYZ whose side is of length 5 cm. Draw the perpendicular on each side from the vertex opposite to it. In how many points do these perpendiculars intersect?
9. Contruct △ ABC such that l(AB) = 7 cm. l(BC) = 8.5 cm. $m\angle B$ = 120°. Draw a perpendicular from the point A on side BC, and measure its length.

§ 4.2 A triangle can be constructed provided three independent conditions about the triangle are given. Given three independent conditions involving sides and angles of a triangle you know how to construct such a triangle. This year we will see how a triangle can be constructed if three independent conditions involving other than angles and sides of a triangle are given.

Some such conditions are studied below.

Constructions of triangles:—

We will see how to construct a triangle

(1) When length of its two sides and an angle not included by them are given.

(2) When its base, sum of the lengths of the two remaining sides and an angle at the base, are given.

(3) When the base, one side and the altitude to the base or the given side, are given.

(4) It's base and the two altitudes from the extremeties of the base are given.

Let us study the above constructions one by one.

(1) **To construct a triangle, in which the lengths of two sides and an angle not included by them, are given.**

Example:— Draw a △ ABC in which l(AB) = 5 cm, l(AC) = 3 cm. and $m\angle B = 30°$

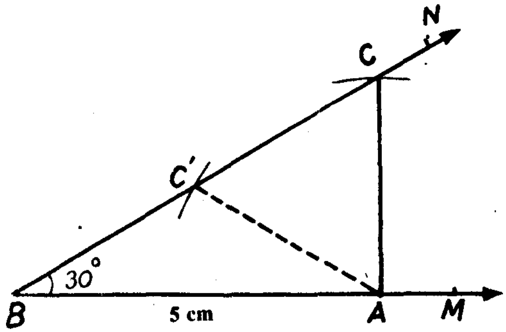


Fig. 4.14

Construction:—

Draw a ray BM. Take point A on it at a distance of 5 cm from the point B. Draw ray BN making an angle of 30° with the ray BA at the point B.

With centre A and radius equal to 3 cm draw an arc of a circle intersecting the ray BN in the point C. Join points A and C. Δ ABC is the required triangle.

Another Δ ABC′ can also be obtained as shown in the figure.]

(2) **To construct a triangle; given its base, sum of the two remaining sides and an angle at the base.**

Example:— construct Δ XYZ in which $l(YZ) = 5$ cm. $l(XY) + l(XZ) = 6.8$ cm and $m\angle Y = 35^\circ$.

Analysis:— In Δ XYZ produce segYX such that Y – X – M and $l(XZ) = l(XM)$. The point X will be on the perpendicular bisector of seg ZM. Hence to construct a ΔXYZ, construct Δ MYZ in which $l(YZ) = 5$ cm. $l(YM) = 6.8$ cm and $m\angle Y = 35^{c}$. To determine the position of the point X, we need to draw a perpendicular bisector of seg ZM

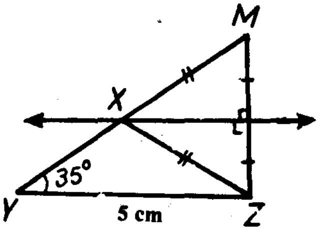


Fig. 4.15

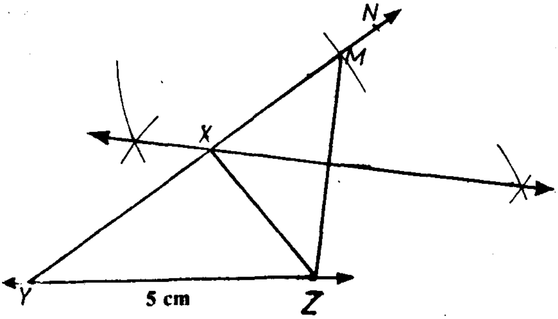


Fig. 4.16

Construction:— Draw a seg YZ of length 5 cms. Draw a ray YN making an angle of 35° with seg YZ at the point Y. With centre Y and radius equal to 6.8 cm draw an arc of a circle interesting the ray YN in point M.

Join points M and Z. Then draw a perpendicular bisector of seg MZ intersecting the ray YN in point X. Join points X and Z. Δ XYZ is the required triangle.

(3) To construct a triangle; given the base, one side and the altitude to the base or the given side.

Example:— Draw Δ XYZ in which $l(XY) = 4$ cm, $l(YZ) = 5$ cm, $l(XM) = 3$ cm where seg XM $\perp$ seg YZ.

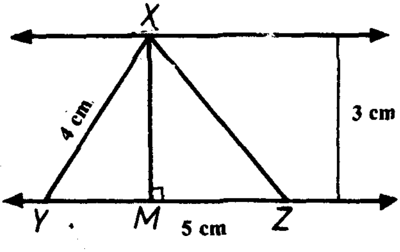


Fig. 4.17

Analysis:— seg YZ is the base and seg XM is the perpendicular on it from the vertex X. $l(XM) = 3$ cm. So X is the point on a line parallel to segYZ at a distance of 3 cm from it.

Hence first draw seg YZ and then draw a line parallel to it at a distance of 3 cm. The point X is on this line at a distance 4 cm from Y.

Construction:— First draw a line and with the help of a compass take a seg YZ of length 5 cm on it. Take any point R on line YZ (Y – Z – R). Draw a seg RN of length 3 cm such that seg RN $\perp$ line YZ. Through the point N draw a line L $\parallel$ line YZ.

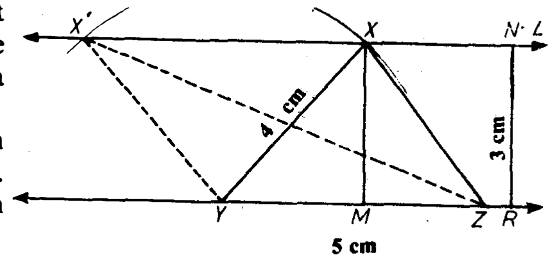


Fig. 4.18

With centre Y and radius equal to 4 cm draw an arc of a circle intersecting the line L in point X. Join points X and Y and points X and Z. Δ XYZ is the required triangle.

(Another Δ X'YZ can also be obtained as shown in the Figure).

(4) To construct a triangle, given its base and the two altitudes from the extremeties of the base.

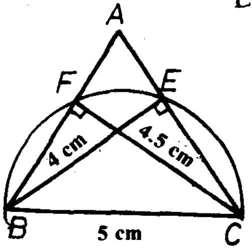


Fig 4.19

Example:—

Draw a Δ ABC in which, $l(BC) = 5$ cm, $l(BE) = 4$ cm. $l(CF) = 4.5$ cm, where seg BE $\perp$ seg AC, and seg CF $\perp$ seg AB.

Analysis:— seg BE $\perp$ seg AC and seg CF $\perp$ seg AB. As $m\angle$ BFC = $m\angle$ BEC = 90° $\angle$ BFC $\cong$ $\angle$ BEC. These angles are substended by seg BC at F and E respectively and have measure 90° each. Points F and E will lie on a semicircle with seg BC as diameter. On this semicircle point F can be located at a distance 4.5 cm from C and E can be located at a distance 4cm from B.

Construction:—

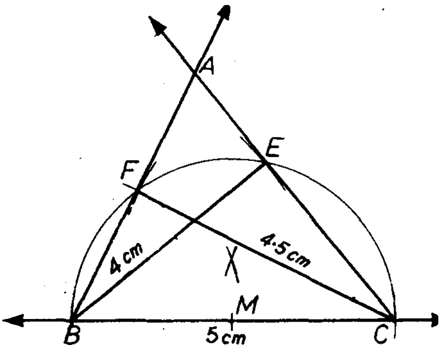


Fig. 4.20

On a line draw a seg BC of length 5cm then bisect seg BC and draw a semicircle with seg BC as a diameter. With centre B and radius equal to 4 cm draw an arc of a circle intersecting the semicircle in point E. Similarly with centre C and radius equal to 4.5 cm draw an arc of a circle intersecting the semicircle in point F. Produce ray BF and ray CE to meet at a point A. Δ ABC is the required triangle.

Exercise — 4.2

1. In Δ XYZ, $l(XY) = 5.6$ cm, $m\angle X = 50^\circ$ and $l(YZ) = 4.3$ cm, construct Δ XYZ.
2. Construct Δ ABC such that $l(BC) = 4.6$ cm. $l(AB) + l(AC) = 6.3$ cm. and $m\angle B = 40^\circ$
3. In Δ XYZ, $m\angle Y = 50^\circ$, $l(YZ) = 4.8$ cm, seg XP ⊥ seg YZ, $l(XP) = 3.2$ cm. Construct Δ XYZ.
4. Contruct Δ ABC such that $l(BC) = 5$ cm, $l(BE) = 4$ cm, $l(CF) = 3.5$ cm, seg BE ⊥ seg AC and seg CF ⊥ seg AB. Find $l(AB)$.

§ 4.3 Constructing a quadrilateral.

It is possible to construct a triangle if three independent elements of the triangle are given. In the same way it is possible to construct a quadrilateral if its five independent elements are given.

1. **To construct a quadrilateral when three sides and two diagonals are given.**

Example:—Construct a □ABCD in which $l(AB) = 5.4$ cm, $l(BC) = 2.7$ cm, $l(AD) = 4.8$ cm, $l(AC) = 6$ cm, $l(BD) = 6.6$ cm.

Analysis:— As all sides of Δ ABC and Δ ABD are known, they can be drawn. Joining points C and D we get the quadrilateral ABCD. To get a convex quadrilateral points C and D must be on the same sides of line AB.

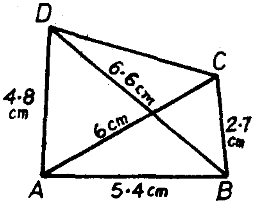


Fig. 4.21

Construction:— On a line draw a seg AB of length 5.4 cm. With centre A and radius equal to 6 cm draw an arc. Similarly with centre B and radius 2.7 cm draw an arc intersecting the first arc in point C. Again with centre A and radius 4.8 cm draw an arc. Similarly with centre B and radius 6.6 cm draw an arc intersecting the first arc in point D such that point D is on the same side of line AB as the point C. Joining points C and D we get the required quadrilateral ABCD.

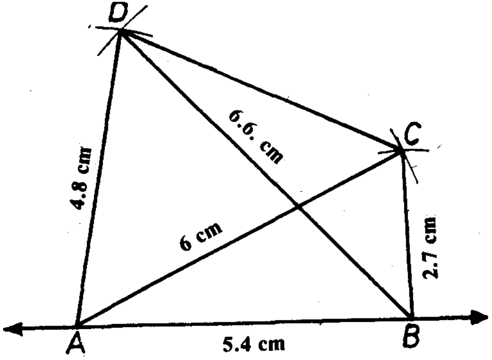


Fig. 4.22

2. **To construct a trapezium, having given two non-parallel sides, one of the parallel sides and an angle at the end point of this side.**

Example:— Construct a trapezium in which *l*(AB) = 6.9 cm *l*(BC) = 3.9 cm, *l*(AD) = 3.3 cm, $m\angle B = 50^\circ$ and seg AB $\parallel$ seg CD.

Construction:— On a line, draw on it a seg AB of length 6.9 cm. Draw a ray BN making an angle of 50° at point B with ray BA. With centre B and radius 3.9 cm draw an arc intersecting the ray BN in point C. Through point C draw a ray CM parallel to seg AB. With centre A and radius 3.3 cm draw an arc intersecting the ray CM in point D. Join points A and D. □ABCD is the required trapezium. (Another trapezium ABCD can also be obtained as shown in the Figure 4.23.)

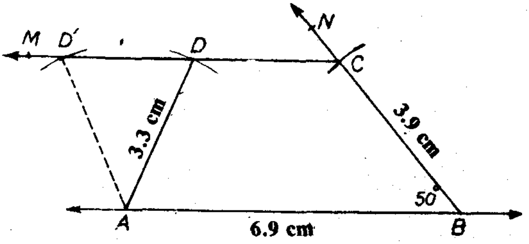


Fig. 4.23

3. **To construct a parallelogram having given its altitude and two of its adjacent sides.**

Example:— Draw a parallelogram ABCD in which *l*(AB) = 6 cm, *l*(AD) = 3.6 cm altitude to the base AB = 3.2 cm.

Construction:—

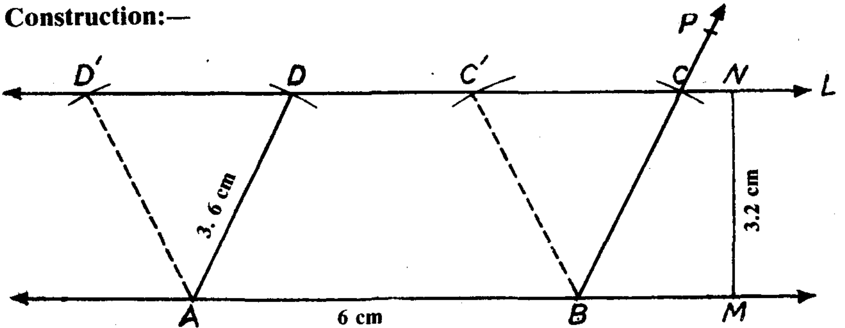


Fig. 4.24

On a line draw a seg AB of length 6 cm. Take any point M on line AB (preferabiy A—B—M). Through point M draw a seg MN of length 3.2 cm. perpendicular to the line AB. Through the point N draw a line L parallel to line AB. With centre A and radius 3.6 cm draw an arc intersecting the line L in points D and D . Join points A and D. (also A and D′.). Through point B draw a ray BP parallel to seg AD, intersecting the line L in point C. □ABCD is the required parallelogram.

Similarly point C′ on line L can be obtained by drawing line parallel to seg AD′ and another parallelogram ABC′D′ is also obtained as shown in the figure.

4. To construct a rectangle, given a side and a diagonal.

Example:—

Draw a rectangle ABCD having given l(AB) = 4.7 cm, seg AC is a diagonal and l(AC) = 6 cm.

Construction:—

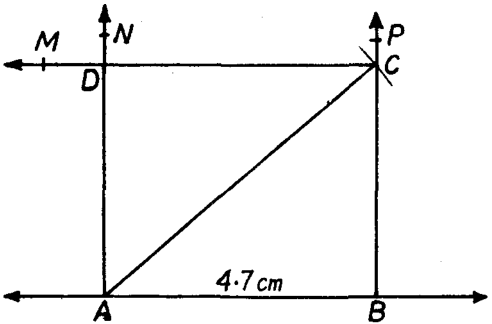


Fig. 4.25

On a line draw a seg AB of length 4.7 cm At point B draw a ray BP ⊥ to the seg AB. With centre A and radius 6 cm, draw an arc intersecting the ray BP in point C. Join points A and C.

Through point A draw a ray AN parallel to ray BP. Similarly through point C draw a ray CM parallel to seg AB. Let the ray CM and ray AN intersect in point D. Then □ABCD is the required rectangle.

5. To construct a rhombus when its two diagonals are given.

Example:— Draw a rhombus ABCD, given that seg AC and seg BD are the diagonals of the rhombus ABCD. l(AC) = 5.2 cm and l(BD) = 3.6 cm.

Analysis:— Diagonals of a rhombus bisect each other at right angles. Hence diagonals AC and BD will bisect each other in point M at right angles. This suggests the following construction.

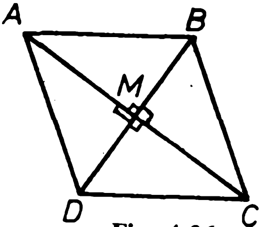


Fig. 4.26

Take a seg AC of lenth 5.2 cm. Draw a line L as a perpendicular bisector of seg AC. Let line L intersect seg AC in point M. With centre M and radius 1.8 cm draw an arc intersecting the line L in points B and D. Join points A,B,C and D in order to form a □ABCD. □ABCD is then, required rhombus.

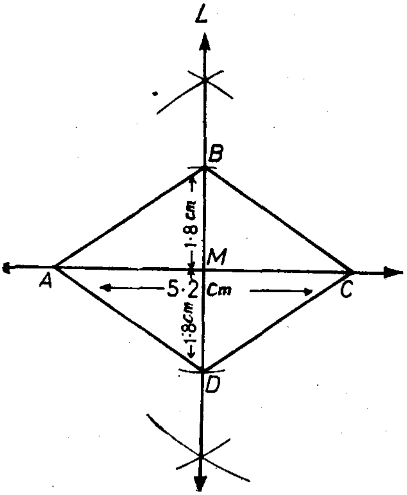


Fig. 4.27

Exercise:— 4.3

1. From the following data draw trapezium ABCD in which l(AD) = 3 cm, l(BC) = 3.2 cm, l(AB) = 5 cm. $m\angle B = 50°$ and seg AB || seg CD.
2. In a prallelogram ABCD, seg DE $\perp$ seg AB. l(DE) = 4.8 cm, l(DC) = 8 cm, l(BC) = 5.3 cm. Draw parallelogram ABCD,
3. In rectangle ABCD, seg AC is the diagonal. l(AC) = 4.7 cm and l(CD) = 4.2 cm, construct the rectange ABCD.
4. Draw the rhombus EFGH in which seg EG and seg FH are the diagonals and l(EG) = 5.7 cm, l(FH) = 7.8 cm Find the length of seg EF.

§ 4.4 Circumcircle of a given triangle:

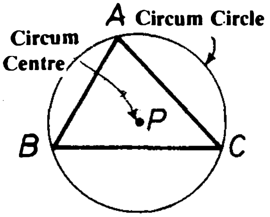


Fig. 4.28

A circle passing through the three vertices of a triangle is called the circumcircle of the given triangle.

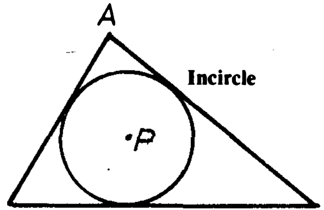


Fig. 4.29

Incircle of a given triangle:

A circle which touches the three sides of a triangle **internally** is called the incircle of the given triangle.

(1) To draw a circumcircle of a given triangle.

Example:— In Δ DEF, l(DE) = 4 cm, l(EF) = 8.6 cm and l(DF) = 6.5 cm. Draw the circumcircle of Δ DEF.

Analyse:— Let Δ DEF be the given triangle and S be the circumcentr S will be equidistant from points D, E and F. So it will lie on the perpendicular bisector of seg DE, seg EF and seg DF. 'S' can be fixed as the point of intersection of perpendicular bisectors of any two sides of Δ DEF.

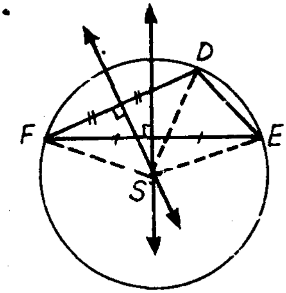


Fig. 4.30

Construction:—Draw a Δ DEF having l(DE) = 4 cm., l(EF) = 8.6 cm., l(FD) = 6.5 cm.

Draw the perpendicular bisectors of seg DF and seg EF. Let them intersect in point S. With centre S, and radius equal to l(DS) draw a cirle. It is the required circumcircle of Δ DEF.

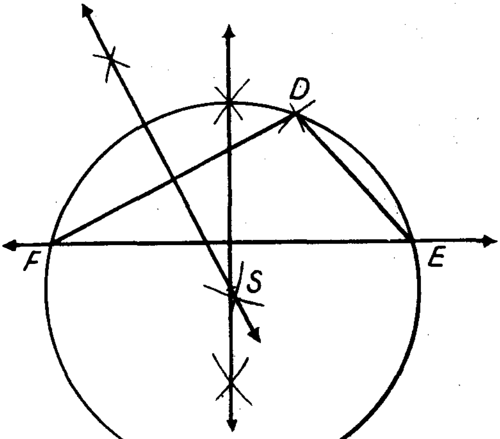


Fig. 4.31

(2) To draw an incircle of a given triangle.

Example:— In △ PQR, l(QR) = 9 cm, $m\angle$ Q = 50°, $m\angle$ R = 47° construct its incircle.

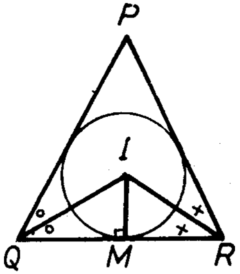


Fig. 4.32

Analysis: Let △ PQR be the given triangle and I be the centre of the incircle. Since I is the point equidistant from seg QR and seg PQ, I lies on the bisector of ∠ Q. It will lie, similarly, on the bisector of ∠ R.

Construction:— Draw a triangle PQR having l(QR) = 9 cm, $m\angle$Q = 50° and m ∠R = 47°. Bisect ∠Q and ∠R. Let the bisectors meet in point I From the point I draw a perpendicular seg IM to the seg QR. With centre I and radius equal to l(IM) draw a circle.

It is the required incircle of the △ PQR.

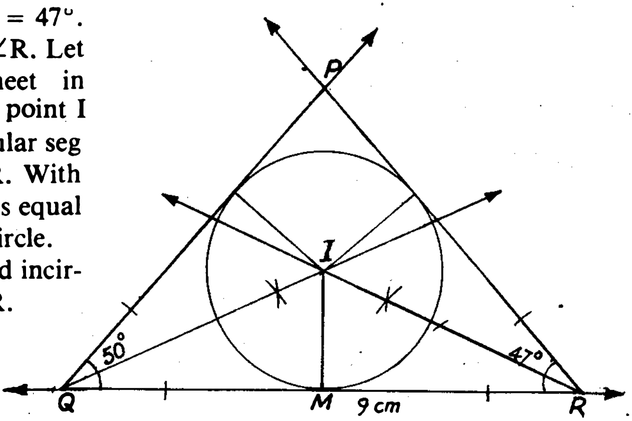


Fig. 4.33

Exercise 4.4

1. Construct an equilateral △ PQR in which l(PQ) = 6 cm. Construct the circum circle of △ PQR, measure the radius.
2. In △ PQR, l(PQ) = l(PR) = 7 cm, l(QR) = 4.5 cm. Draw a circumcircle of △ PQR. Measure its radius.
3. In △ ABC, l(AB) = 6.5 cm, l(BC) = 8.3 cm, $m\angle$ B = 90°. Draw incircle of △ ABC.
4. In △ ABC, l(AB) = 6 cm, l(BC) = 8 cm, m ∠B = 135°. Draw incircle of △ ABC.
5. In equilateral △ ABC, l(AB) = 6.5 cm Draw the incircle and circumcircle of △ ABC. Find the radii and ratio of the length of the two radii.

§ 4.5 Tangent of a circle:

(1) To draw a tangent line to a given circle at a point on it.

Example:—

Draw a tangent line to the circle of radius 2.4 cm at a point P on it.

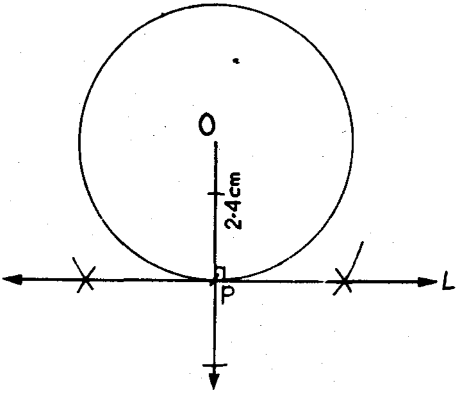


Fig. 4.34

Analysis:— A perpendicular line to the radius at its outer end is tangent to the circle at that point.

Construction:— With centre O and radius 2.4 cm draw a circle. Let P be the given point on it. Draw ray OP. At point P draw the line L perpendicular to the ray OP. Line L is the required tangent line.

(2) To draw two tangent lines to a circle through a given point out side the circle.

Example:— O is the centre of a circle of radius 3 cm. P is the point outside the circle such that $l(OP) = 7.5$ cm. Draw the tangent lines to the circle from the point P.

Analysis:— Let line PQ and line PR be the two tangent lines, drawn from the point P to the circle with centre O. Suppose Q and R are the points of contact. Since tangent line is perpendicular to the radius at the point of contact, $m\angle OQP = m\angle ORP = 90°$. So points Q and R lie on the circle, drawn on seg OP as diameter.

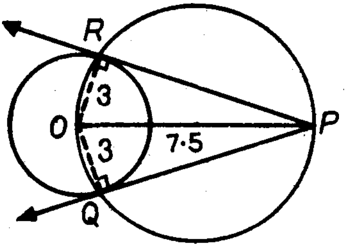


Fig. 4.35

Construction:— With centre O draw a circle of radius 3 cm. Let P be any point outside this circle such that l(OP) = 7.5 cm. Bisect seg OP. Draw a circle with seg OP as a diameter cutting the circle with centre O in points Q and R. Join points P and Q and also points P and R. The line PQ and line PR are the required tangent lines from P to the given circle.

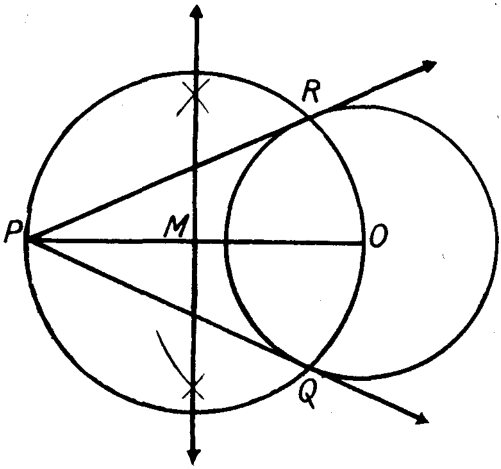


Fig. 4.36

Exercise 4.5

1. Draw a circle of radius 3.5 cm. Take a point M at a distance of 7.5 cm from the centre. Draw the tangent lines to the circle from point M.

2. Draw a circle with centre O and radius 4 cm. Take a chord AB of length 6 cm. Draw the two tangent lines at points A and B intersecting at point P. Find l(OP).
3. Take l(AP) = 8 cm. Draw a circle with point A as a centre and of radius 3.5 cm. From point P draw seg PL and seg PM, the tangent line segments to the given circle. Find l(PM) and l(PL).
4. Draw a circle of radius 3.5 cm. Take any point P on it and draw a tangent line to the circle at the point P.

Miscellaneous Exercise: 4

1. In. Δ ABC, l(BC) = 5.3 cm, l(AC) = 6.2 cm and $m\angle A = 42°$. Draw the Δ ABC and measure the length of the side AB.
2. In Δ ABC, l(BC) = 5.9 cm, altitude AD = 4.1 cm and l(AB) = 4.8 cm . Construct the Δ ABC and measure $\angle$ B.
3. In □PQRS, l(PQ) = 5.2 cm, l(QR) = 3.2 cm, l(SP) = 4.6 cm, l(PR) = 6.2 cm and l(QS) = 6.8 cm. Construct the □PQRS and measure the length of the side RS.

4. In $\triangle$ LMN, $l(MN) = 5.5$ cm, $m\angle M = 40°$, $l(LM) + l(LN) = 11.3$ cm. Construct the $\triangle$ LMN and measure the length of the sides LM and LN.
5. In $\triangle$ RST, $l(RS) = 5.6$ cm, altitude RP $= 3.7$ cm and altitude SQ $= 4$ cm. Draw the $\triangle$ RST and measure $\angle S$.
6. In a rectangle DEFG, $l(DE) = 6.2$ cm, $l(DF) = 7$ cm. Draw the rectangle DEFG and measure the length of the side EF.
7. In $\triangle$ ABC, $l(BC) = 6.2$ cm, $l(CA) = 5.1$ cm, and $l(AB) = 5.5$ cm. Draw the $\triangle$ ABC, and then construct the circumcircle of that triangle. Measure its radius.
8. $\square$PQRS is a rhombus, $l(PR) = 6.4$ cm, $l(QS) = 8.2$ cm. Construct the $\square$PQRS and measure its side.
9. Draw a circle of readius 3.2 cm. Take any point P on it. Draw a tangent to the circle at the point P.
10. In $\triangle$ ABC, $l(BC) = 6.2$ cm, $m\angle C = 50°$ and $l(CA) = 5.2$ cm, Construct the incircle of the $\triangle$ ABC. Measure the radius of the incircle.
11. Draw circle with centre P and radius 3.2 cm. Take a point Q, 8.2 cm away from the point P. Draw tangent segments through point Q, touching the circle in points A and B. Measure the lengths of the tangent segments.
12. $\square$XYZW is a parallelogram. $l(XY) = 5.8$ cm. The height of the parallelogram XYZW for the base XY is 3 cm. $l(YZ) = 3.5$ cm. Construct the $\square$XYZW. Measure $\angle XYZ$.
13. In $\triangle$ ABC, $l(BC) = 6$ cm, altitude AD $= 4.7$ cm, $l(AB) = 5.3$ cm. Construct the $\triangle$ ABC and measure $\angle BAC$.
14. $\square$ABCD is a trapezium, having seg AB $\parallel$ seg DC. $l(AB) = 6.1$ cm, $m\angle B = 40°$, $l(AD) = 3.6$ cm. $l(BC) = 4$ cm, Draw the $\square$ABCD and measure $\angle A$.
15. Draw an equilateral $\triangle$ ABC, having its side of length 5 cm. Draw the incircle of the $\triangle$ ABC and measure its radius.

□ □ □

Chapter 5

SIMILARITY

§ 5.1 Ratio and Proportion:

You have studied ratio and proportion in your previous class. These two notions are closely related to the notion of similarity of geometrical figures. Therefore, before we start studying similarity in geometrical figures let us recall some ideas about ratio and proportion in relation to geometry.

We know that between the vertices of two traingles there are in all six one-to-one correspondences. Because of any one of these, the sides and angles of the two triangles get paired. These are called corresponding sides and corresponding angles. Given a one-to-one correspondence between the vetices of two triangles, if the ratio of the lengths of the corresponding sides are equal in value the sides of the two triangles are said to be proportional.

Given △ ABC, △ DEF and ABC ⟷ DEF, fill in the following blanks properly to show which sides and angles get paired.

side AB ⟷ side _ _ _ _ ; ∠ _ _ _ _ ⟷ ∠D
side _ _ _ _ ⟷ side EF ; ∠ B ⟷ _ _ _ _
side _ _ _ _ ⟷ side _ _ _ _ ; ∠ C ⟷ ∠ _ _ _ _

Given △ ABC, △ DEF and ABC ⟷ DEF. If the following holds,

$\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)}$, we say that the sides of the two triangles are proportional.

Coversely, if given ABC ⟷ DEF and the corresponding sides of the two triangles are proportional, then $\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)}$ holds.

In short, to say that corresponding sides of the two triangles are proportional is the same as saying that the ratios of the lengths of the corresponding sides are equal in value.

Example 1:— Given ABC ⟷ PQR and additional information as given in the figure 5.1 state whether the corresponding sides of the two triangles are proportional.

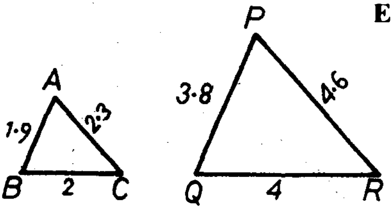


Fig. 5.1

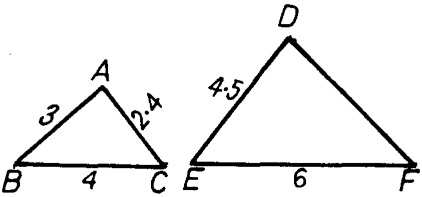


Fig. 5.2

Example 2:— It is given that under ABC ⟷ DEF, the corresponding sides of the two triangles are proportional. From additional information as given in fig. 5.2., find l(DF).

Example 3 :— From the information as given in the fig. 5.3, state whether under XYZ ⟷ UVW the corresponding sides of △ XYZ and △ UVW are proportional.

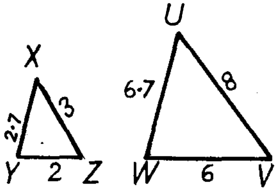


Fig. 5.3

§ 5.2 Similarity:

Below some figures are given in pairs.

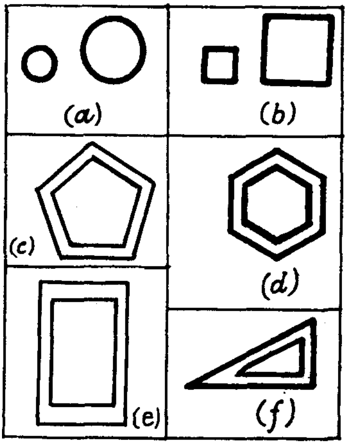


Fig. 5.4

Since figures in each pair differ in respect of size, they cannot be congruent to each other. Although the figures in each pair are not congruent, they very much look alike. Such figures are said to be similar to each other.

Like congruence, similarity plays a very important role in geometry. Even in our day-to-day life we have to use this notion on various occasions. Before constructing a house we have to prepare its map. The map of a country enables us to study its geography. By taking a real photograph of an enemy country it is possible to know the locations of centrės of industrial production and important places of military establishments. The map or a photograph becomes useful on the basis of similarity.

If we study the reasons as to why actual country and its map or its photograph are similar, the first thing that strikes us is that all sides are enlarged or are diminished proportionally. Secondly the angle between the two sides in real situation is congruent to the corresponding angle in the map. The map of a rectangular room has to be rectangular to appear similar. The angle between any two roads in a city is congruent to the corresponding angle in their map or photograph.

By merely looking at the following pairs of triangles we can decide that the triangles in each pair are not similar to each other.

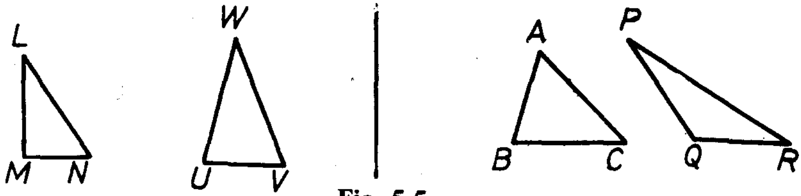


Fig. 5.5

If we study varius pairs of similar and non-similar triangles we observe that the triangles do not look similar if either one of the following conditions holds.

(1) If corresponding angles of any one of the pairs are not congruent, or

(2) If the corresponding sides of two triangles are not proportional.

Therefore, given ABC $\longleftrightarrow$ DEF, for $\triangle$ ABC, $\triangle$ DEF to be similar, it is necessary that following conditions be satisfied.

(i) $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$.

(ii) $\dfrac{l(AB)}{l(DE)} = \dfrac{l(BC)}{l(EF)} = \dfrac{l(CA)}{l(FD)}$.

In other words, for two triangles to be similar it is necessary that,

(1) corresponding angles be congruent

(2) corresponding sides be proportional.

Thus, given ABC $\longleftrightarrow$ DEF, to say that $\triangle$ ABC is similar to $\triangle$ DEF is to mean that,

$$\angle A \cong \angle D,\ \angle B \cong \angle E,\ \angle C \cong \angle F, \text{ and}$$

$$\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)}.$$

Conversely, if it is given that under ABC $\longleftrightarrow$ DEF

(i) $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$ and

(ii) $\dfrac{l(AB)}{l(DE)} = \dfrac{l(BC)}{l(EF)} = \dfrac{l(CA)}{l(FD)}$ hold, then it means that $\triangle$ ABC is similar to $\triangle$ DEF.

§ 5.3 Similarity sign and some properties of similarity:—

To indicate similarity we will use the sign $\sim$.

$\triangle$ ABC $\sim$ $\triangle$ DEF will then mean that under ABC $\longleftrightarrow$ DEF, $\triangle$ ABC is similar to $\triangle$ DEF.

As in case of congruence, the relation of similarity has the following three properties:

(i) $\Delta ABC \sim \Delta ABC$.

Every triangle is similar to itself.

(ii) If $\Delta ABC \sim \Delta DEF$ then $\Delta DEF \sim \Delta ABC$.

If one triangle is similar to a second triangle this second triangle is similar to the first.

(iii) If $\Delta ABC \sim \Delta DEF$ and $\Delta DEF \sim \Delta XYZ$ then $\Delta ABC \sim \Delta XYZ$.

If one triangle is similar to a second triangle and this second triangle is similar to a third tringle, then the first triangle is similar to the third.

Congruence and similarity have a few properties in common. Then in what respects exactly do these relations differ?

In order that the two given traingles be either congruent or similar it is necessary in both cases that corresponding pairs of angles be congruent. To obtain congruence it is further necessary that corresponding sides be also congruent, wheareas to obtain similarity it is further necessary that ratios of lengths of corresponding sides are equal in value. If this proportion is 1:1, we get congruence; if not there is still similarity.

When the triangles are not congruent but are similar, the proportion of lengths of corresonding sides is either greater than 1:1 or less than 1:1.

From the above discussion it is clear that when the two traingles are congruent they are in fact similar. However if it is given that two triangles are similar, they may or may not be congruent.

Exercise 5.1

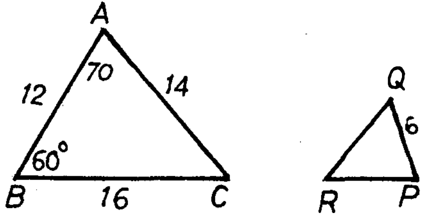


Fig. 5.6

1. From the information supplied in figs 5.6, 5.7 give the information asked.

(i) If $ABC \longleftrightarrow PQR$ and $\Delta ABC \sim \Delta PQR$, then, $l(PR) = ?$, $l(QR) = ?$, $m\angle P = ?$, $m\angle Q = ?$, $m\angle R = ?$.

(ii) If $\Delta XYZ \sim \Delta LMN$ then, $l(LM) = ?$, $l(MN) = ?$, $m\angle M = ?$.

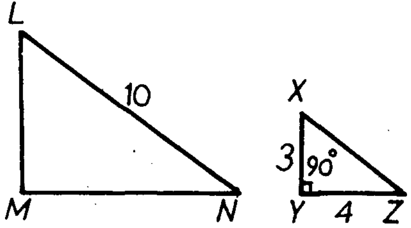


Fig. 5.7

2. In fig. 5.8, information regarding the △ ABC and △ PNM is given. With the help of the given information fill in properly the blanks given.

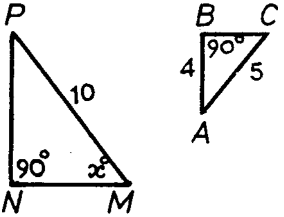


Fig. 5.8

If △ ABC ∾ △ PNM, then

m ∠ A = ——

m ∠ C = ——

m ∠ P = ——

l(MN) = ——

l(PN) = ——

l(BC) = ——

3. In fig. 5.9, △ ABC ∾ △ QPR. Fill in properly the blanks given.

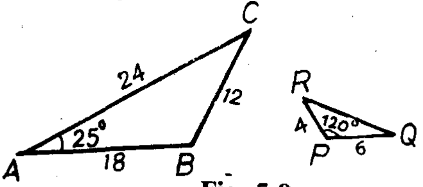


Fig. 5.9

(i) *l*(QR) = ——

(ii) m ∠ Q = ——

(iii) m ∠ B = ——

(iv) m ∠ C = ——

4. In the fig. 5.10 if △ ABC ∾ △ ADE, *l*(AD) = 5. *l*(AE) = 6, *l*(BC) = 12, *l*(AB) = 15 then find *l*(AC) and *l*(DE).

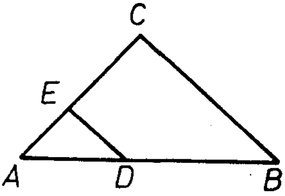


Fig. 5.10

§ 5.4 Some important theorems about areas of triangles.

Theorem: Areas of two triangles are proportional to the products of base and respective height of the two triangles.

In other words ratio of areas of two triangles is equal to the ratio of product of base and the respective height of the two triangles.

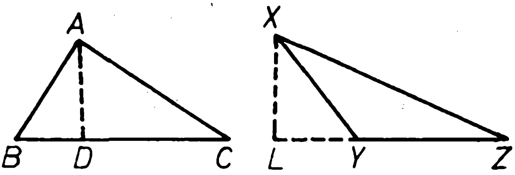


Fig. 5.11

Given: In △ ABC, seg AD ⊥ seg BC, B-D-C and in △ XYZ, seg XL ⊥ line YZ, L-Y-Z.

To prove: $\dfrac{A(\triangle ABC)}{A(\triangle XYZ)} = \dfrac{l(BC) \times l(AD)}{l(YZ) \times l(XL)}$

Proof:
$$\left.\begin{aligned} A(\triangle ABC) &= \tfrac{1}{2}\, l(BC) \times l(AD) \\ A(\triangle XYZ) &= \tfrac{1}{2}\, l(YZ) \times l(XL) \end{aligned}\right\} \text{by using relation for the area of a triangle}$$

$$\left.\begin{aligned} \therefore\ \frac{A(\Delta ABC)}{A(\Delta XYZ)} &= \frac{l(BC) \times l(AD)}{l(YZ) \times l(XL)} \\ \text{or}\quad & \\ \frac{A(\Delta ABC)}{l(BC) \times l(AD)} &= \frac{A(\Delta XYZ)}{l(YZ) \times l(XL)} \end{aligned}\right\} \text{\_\_\_\_(1)}$$

Now suppose that these two triangles are of equal heights. Then, $l(AD) = l(XL)$ in (1) gives

$$\frac{A(\Delta ABC)}{A(\Delta XYZ)} = \frac{l(BC)}{l(YZ)}, \text{ or } \frac{A(\Delta ABC)}{l(BC)} = \frac{A(\Delta XYZ)}{l(YZ)}$$

Therefore, triangles of equal heights have areas proportional to their bases.

Again suppose that the triangles have equal bases. Then, $l(BC) = l(YZ)$ in (1) gives

$$\frac{A(\Delta ABC)}{A(\Delta XYZ)} = \frac{l(AD)}{l(XL)} \text{ or } \frac{A(\Delta ABC)}{l(AD)} = \frac{A(\Delta XYZ)}{l(XL)}$$

Therefore, triangles of equal bases have their areas proportional to their heights.

Exercise 5.2

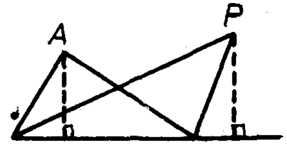


Fig. 5.12

1. In fig 5.12, seg AE ⊥ seg BC, B-E-C and seg PM ⊥ seg BC, E-C-M. If $l(AE) = 12$ and $l(PM) = 16$ then,

$$\frac{A(\Delta ABC)}{A(\Delta PBC)} = ?$$

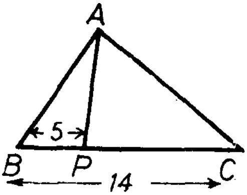


Fig. 5.13

2. In fig 5.13 B-P-C. $l(BC) = 14$ and $l(BP) = 5$. There are three triangles of equal heights. Find them and fill in the blanks.

$\frac{A(\Delta ABP)}{A(\Delta APC)} =$ ______ $\qquad$ $\frac{A(\Delta ABP)}{A(\Delta ABC)} =$ ______

$\frac{A(\Delta APC)}{A(\Delta ABC)} =$ ______

3 In fig. 5.14, line L || line M. From the figure answer the following questions.

(i) Name three triangles having base seg AB.

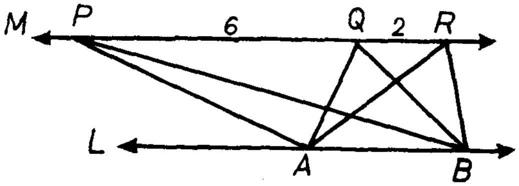


Fig. 5.14

(ii) What conclusion can you draw about the height and areas of triangles on segment AB?

(iii) Are triangles, Δ APQ, Δ BPQ, Δ BQR, Δ BPR and Δ APR of the same height?

(iv) Write the values of the following ratios:

(1) $\frac{A(\Delta APQ)}{A(\Delta BPR)}$ (2) $\frac{A(\Delta AQR)}{A(\Delta APR)}$ (3) $\frac{A(\Delta BPR)}{A(\Delta AQR)}$ (4) $\frac{A(\Delta APR)}{A(\Delta BPR)}$

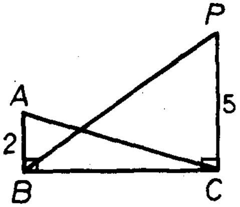


Fig. 5.15

4. In fig. 5.15, $m\angle ABC = m\angle PCB = 90°$. $l(AB) = 2$, $l(PC) = 5$.

Find $\frac{A(\Delta ABC)}{A(\Delta PBC)}$.

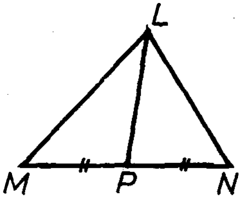


Fig. 5.16

5. In fig 5.16, segment LP is the median of Δ LMN. Show that A(Δ LMP) = A(Δ LPN)

[**A median of a triangle divides the triangle into two triangles of equal areas.**]

6. From the fig. 5.17 select at least one pair of triangles of given description. Line L || Line M.

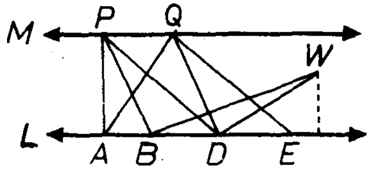


Fig. 5.17

(i) Triangles of equal bases and different heights.

(ii) Triangles of equal heights and different bases.

(iii) Triangles of equal bases and equal heights.

7. In fig. 5.18 Line L || Line M. Line M is a number line. $l(\text{AB}) = 6$, $l(\text{BC}) = 2$.

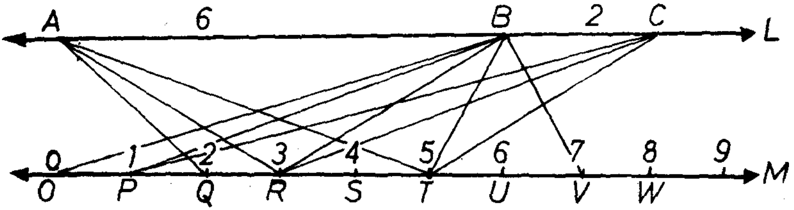


Fig. 5.18

From the figure and the given information fill in properly the blanks below.

(i) $\dfrac{A(\Delta\,\text{BPT})}{A(\Delta\,\text{PCR})} = \_\_\_\_$ (ii) $\dfrac{A(\Delta\,\text{QAR})}{A(\Delta\,\text{PCT})} = \_\_\_\_$

(iii) $\dfrac{A(\Delta\,\text{RBO})}{A(\Delta\,\text{QAT})} = \_\_\_\_$ (iv) $\dfrac{A(\Delta\,\text{PBT})}{A(\Delta\,\text{RBV})} = \_\_\_\_$

(v) $\dfrac{A(\Delta\,\text{ABR})}{A(\Delta\,\text{PBV})} = \_\_\_\_$. (vi) $\dfrac{A(\Delta\,\text{ART})}{A(\Delta\,\text{OBV})} = \_\_\_\_$

8. Two triangles have equal areas. Their bases are in the ratio 1:3. Find the ratio of their heights.

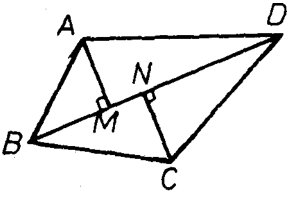


Fig. 5.19

9. On the diagonal BD of a quadrilateral ABCD seg AM and seg CN are perpendicular. Then,

$$\frac{A(\Delta\,\text{ABD})}{A(\Delta\,\text{CBD})} = \frac{l(\text{---})}{l(\text{---})}.$$

If □ABCD is a parallelogram, $l(\text{AM}) = 5$, find $l(\text{CN})$.

10. Areas of two triangles of equal heights are in the ratio 2:3. Base of the smaller triangle is 7. Find the corresponding base of the other triangle.

11. In the figure 5.20, $m\angle\,\text{DEF} = 90^\circ$, seg EP $\perp$ seg DF. D-P-F, then show that,

$$\frac{l(\text{EF})}{l(\text{EP})} = \frac{l(\text{DF})}{l(\text{DE})}$$

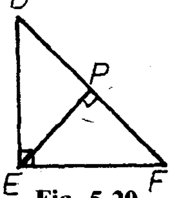


Fig. 5.20

§ 5.5 Three parallel lines and their intercepts:

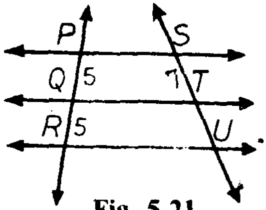


Fig. 5.21

In figure 5.21 three parallel lines and their two transversals are shown. From the figure, $l(TU) = ?$

By filling the blanks below properly indicate the property which you have used in solving the above problem, about three parallel lines and the intercepts.

If three parallel lines make congruent intercepts on a transversal then it makes ——————— intercepts on any other transversal.

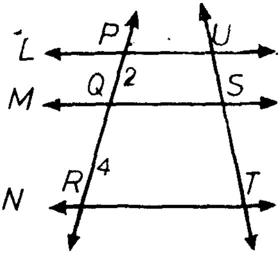


Fig. 5.22

Now study the fig. 5.22. In it are given three parallel lines which do not have congruent intercepts on any tranversal. But in the figure seg QR has twice the length of seg PQ. It is natural to feel that this property holds true in respect of any other transversal. Therefore, $l(ST)$ appears to be twice $l(US)$.

In the previous fig. 5.21 the ratio 1:1 between the lengths of the intercepts remains constant. Our mind feels that the ratio 2:1 remains constant in respect of lengths of intercepts in fig. 5.22. But this property requires proof. To be able to supply its proof, we need to study the basic theorem on proportionality. First let us, therefore, get acquainted with some concepts.

§ 5.6 Division point of a segment and its uniqueness:

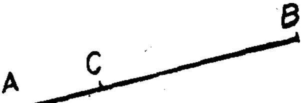


Fig. 5.23

On segment AB if C is a point such that A-C-B, we say that C divides segment AB internally in the ratio $l(AC):l(CB)$. The ratio $\frac{l(AC)}{l(CB)}$ is known as the ratio of internal division.

If, however, the point C is on the line AB such that A-B-C or C-A-B then C is said to divide the seg AB externally in the ratio $l(AC):l(CB)$. The ratio $\frac{l(AC)}{l(CB)}$ is known as the ratio of external division. If A-C-B, C is known as the point of internal division and if A-B-C or C-A-B, it is known as the point of external division.

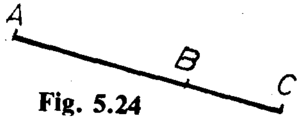


Fig. 5.24

The point of division (whether internal or external) of a given segment dividing the segment in a given ratio is unique.

We will not attempt to prove the above statement. One should bear in mind, however, that a proof of this statement can be given.

§ 5.7 Basic Proportionality theorem:

If a line parallel to a side of a triangle intersects the other two sides in two different points then it divides these sides proportionally.

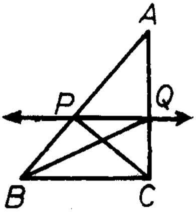


Fig. 5.25

In △ ABC, line PQ intersects seg AB and seg AC in distinct points P and Q respectively. seg PQ || seg BC. To establish the above theorem we will have to show that $\frac{l(AP)}{l(PB)} = \frac{l(AQ)}{l(QC)}$.

Because of seg PC, △ ABC gets divided into △ APC and △ BPC of equal heights.

$$\therefore \frac{A(\triangle APC)}{A(\triangle BPC)} = \frac{l(AP)}{l(PB)} \quad \text{---------- (I)}$$

Similarly because of seg BQ, △ ABC gets divided into △ ABQ and △ CBQ of equal heights.

$$\therefore \frac{A(\triangle ABQ)}{A(\triangle CBQ)} = \frac{l(AQ)}{l(CQ)} \quad \text{---------- (II)}$$

Now, △ BPQ and △ CPQ stand on the same base segment PQ and between the same parallels and so have equal areas. To these two triangles we adjoin △ APQ to get △ ABQ and △ CPA.

$$\therefore A(\triangle APC) = A(\triangle ABQ) \quad \text{---------- (III)}$$

Also, △ BPC and △ BQC stand on the same base segment BC and between the same parallels.

$$\therefore A(\triangle BPC) = A(\triangle BQC) \quad \text{---------- (IV)}$$

From (III) and IV)

$$\frac{A(\triangle APC)}{A(\triangle BPC)} = \frac{A(\triangle ABQ)}{A(\triangle BQC)} \quad \text{---------- (V)}$$

∴ From (I), (II) and (V),

$$\frac{l(AP)}{l(BP)} = \frac{l(AQ)}{l(CQ)} \quad \text{---------- (VI)}$$

Example 1: In fig. 5.26, P and Q are points of internal division of seg AB and seg AC respectively and seg PQ || seg BC. Show that

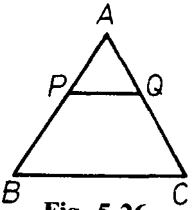


Fig. 5.26

(i) $\frac{l(BP)}{l(AP)} = \frac{l(CQ)}{l(AQ)}$ (ii) $\frac{l(AB)}{l(AP)} = \frac{l(AC)}{l(AQ)}$ (iii) $\frac{l(BP)}{l(AB)} = \frac{l(CQ)}{l(AC)}$

While proving the basic proportionality theorem we had assumed that P and Q were the points of internal division. But the theorem can be proved for the case when both P and Q are points of external division of seg AB and seg AC, for in that case, in Δ APQ the points B and C become points of internal division.

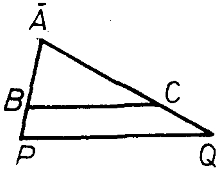


Fig. 5.27

§ 5.8 **Converse of basic proportionality theorem:**

If in Δ ABC points P and Q divide seg AB and seg AC proportionally then seg PQ || seg BC.

Given: In Δ ABC $\frac{l(AP)}{l(PB)} = \frac{l(AQ)}{l(QC)}$.
A-P-B and A-Q-C.
To prove: seg PQ || seg BC.
Proof: Suppose that a line through P parallel to seg BC intersects seg AC in T. Then, by basic proportionality theorem,

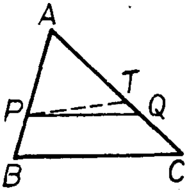


Fig. 5.28

$$\frac{l(AP)}{l(PB)} = \frac{l(AT)}{l(TC)} \quad \text{--------} \quad (I)$$

$$\text{But, } \frac{l(AP)}{l(PB)} = \frac{l(AQ)}{l(QC)} \quad \text{-------} \quad (\text{Given})$$

$$\therefore \frac{l(AT)}{l(TC)} = \frac{l(AQ)}{l(QC)} \quad \text{--------} \quad (II)$$

This shows that there are two points T and Q which divide seg AC in the same ratio. But this is not possible. Therefore, T and Q coincide.

But seg PT || seg BC
∴ seg PQ || seg BC.

Exercise 5.3

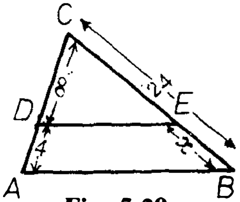


Fig. 5.29

1. In fig 5.29 A-D-C, C-E-B. seg DE || seg AB. $l(AD) = 4$, $l(DC) = 8$, $l(BC) = 24$, $l(EB) = x$.

(i) $\frac{l(AD)}{l(DC)} = \frac{4}{\square}$, (ii) $\frac{l(EB)}{l(EC)} = \frac{x}{\square}$ (iii) $\frac{4}{\square} = \frac{x}{\square}$.

Fill in the blanks above and find the value of x.

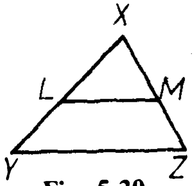


Fig. 5.30

2. In fig. 5.30, X-L-Y, X-M-Z. seg. LM || seg YZ, $l(XY) = 10$, $l(LY) = 6$, $l(XZ) = 25$, $l(MZ) = t$.

Now by filling in the blanks properly determine the value of t.

$\frac{l(YL)}{l(XY)} = \frac{6}{\square}, \frac{l(MZ)}{l(ZX)} = \frac{t}{\square}, \frac{6}{\square} = \frac{t}{\square}$

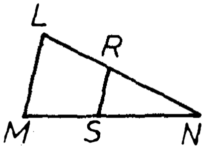


Fig. 5.31

3. In fig. 5.31, seg SR || seg LM. L-R-N, M-S-N, $l(RN) = 8$, $l(LN) = 18$, $l(MS) = 6$, $l(SN) = p$.

By filling in the blanks properly determine the vlaue of p.

$l(LR) = \square$

$\frac{l(LR)}{l(RN)} = \frac{\square}{\square} = \frac{\square}{p} = \frac{l(MS)}{l(SN)}$ $\therefore p = \square$

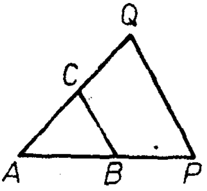


Fig. 5.32

4. In the fig. 5.32, A-B-P, A-C-Q, $l(AB) = 7$, $l(AC) = 8$, $l(AP) = 14$, seg BC || seg PQ. Find $l(AQ)$ and $l(CQ)$.

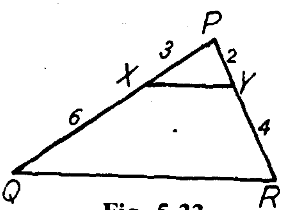


Fig. 5.33

5. From the information given in the fig. 5.33, state whether seg XY || seg QR.

6. In fig. 5.34 if seg DE is to be parallel to seg AB, what should be $l(EC)$?

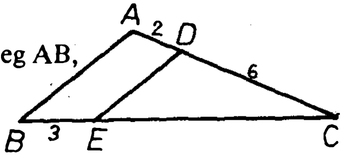


Fig. 5.34

§ 5.9 Three parallel lines and a transversal:

Now we are in a position to answer satisfactorily the question posed earlier in article 5.5 in respect of three parallel lines and intercepts on transversals made by them. L, M and N are three parallel lines and X and Y are their transversals. Line X intersects lines L,M and N in P,Q and R respectively. Line Y intersects lines L,M and N in points E, F and G respectively.

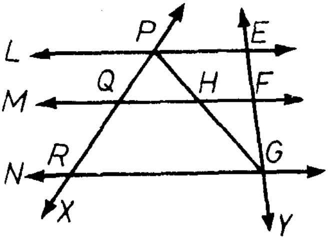


Fig. 5.35

To prove: $\frac{l(PQ)}{l(QR)} = \frac{l(EF)}{l(FG)}$

Construction: Join points P and G and let seg PG intersect line M in H.

Proof: In Δ PRG, line M || seg RG, intersects seg PR and seg PG in distinct points Q and H respectively.

∴ by using basic proportionality theorem we have,

$$\frac{l(PQ)}{l(QR)} = \frac{l(PH)}{l(HG)} \qquad \text{-------(I)}$$

Similarly, in Δ EGP, seg HF || seg PE intersects seg PG and seg EG in distinct points H and F respectively.

∴ by using basic proportionality theorem we have,

$$\frac{l(PH)}{l(HG)} = \frac{l(EF)}{l(FG)} \qquad \text{-------(II)}$$

from (I) and (II) we have,

$$\frac{l(PQ)}{l(QR)} = \frac{l(EF)}{l(FG)} \qquad \text{-------(III)}$$

The ratio of the intercepts made by a transversal on three parallel lines equals the ratio of the corresponding intercepts made by any other transversal on the same parallel lines.

Exercise 5.4

1. In figures 5.36 and 5.37 below, three parallel lines are shown. Lengths of intercepts made by transversals are also shown in the figures. From this information find the value of x.

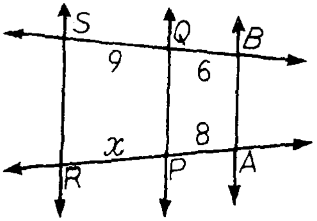


Fig. 5.36

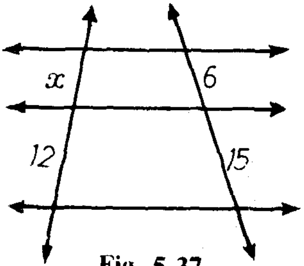


Fig. 5.37

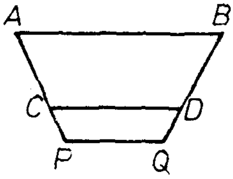


Fig. 5.38

2. In the fig. 5.38 seg AB || seg CD || seg PQ *l*(CP) = 18, *l*(QD) = 15, *l*(BD) = 35, *l*(AP) = ?

3. In fig. 5.39 seg PA, seg QB, seg RC and Seg SD are ⊥ to seg AD.
l(AB) = 60, *l*(BC) = 90, *l*(CD) = 120, *l*(PS) = 360, find *l*(PQ), *l*(QR), *l*(RS).

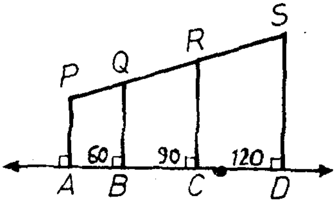
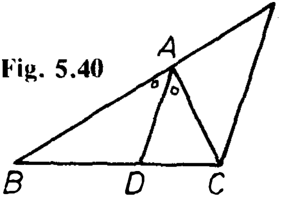


Fig. 5.39

Fig. 5.40



4. Theorem on angle bisector of a triangle.
In fig. 5.40, seg AD is the bisector of ∠ BAC of Δ ABC. B-D-C.
seg CE || seg AD. B-A-E.
Now fill in the gaps below.

∠ BAD ≅ ∠ ______ (corresponding angles) ————————(1)

∠ DAC ≅ ∠ ______ (alternate angles) ————————(2)

But, ∠ BAD ≅ ∠ DAC (given) ————————————(3)

∴ from (1), (2) and (3),

∠ __________ ≅ __________ ————————(4)

In Δ AEC, seg ______ ≅ seg ________ ————————(5)

In Δ BEC, seg AD || seg CE.

∴ by basic proportionality theorem,

$$\frac{l(BA)}{l(AE)} = \frac{l(BD)}{l(\_\_\_)} \text{ ------------(6)}$$

But, from (5), $l(AE)$ = ------------(7)

$$\therefore \frac{l(BA)}{l(AC)} = \frac{l(BD)}{l(DC)} \text{ ------------ (8)}$$

{ Angle bisector of a triangle divides the side opposite to the angle proportional to its sides. }

The above theorem is also true about the bisector of an exterior angle of a triangle.

In the fig. 5.41 ray AD is the bisector of an exterior ∠ CAP of Δ ABC

To prove: $\frac{l(AB)}{l(AC)} = \frac{l(BD)}{l(DC)}$

Let A-X-B, and let seg CX || seg AD.

show seg AX ≅ seg AC

Now, in Δ BAD, seg CX || seg AD.

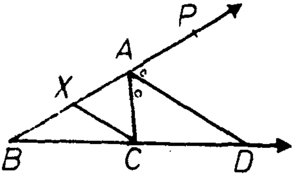


Fig. 5.41

$$\frac{l(AB)}{l(AX)} = \frac{l(BD)}{l(DC)}$$

$$\therefore \frac{l(AB)}{l(AC)} = \frac{l(BD)}{l(DC)}$$

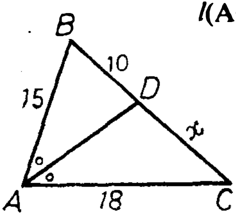


Fig. 5.42

5. In fig. 5.42 seg AD bisects the angle A. From the information given in the figure, find the value of x.

§ 5.10 Necessary and sufficient conditions of similarity:

We have seen that for two triangles to be similar it is necessary that corresponding sides be proportional and corresponding angles be congruent. Even if one of these conditions is broken the triangles are not similar. Now by fulfilling some out of the necessary conditions if the remaining conditions also get fulfilled then we could say that those some conditions would be sufficient to ensure similarity.

Exactly similar situations had arisen when we studied congruence. In order that two triangles become congruent it is necessary that six conditions be satisfied; their corresponding sides and angles have got to be congruent. But we had seen that if out of the six conditions some group of three conditions is satisfied then the remaining three automatically get satisfied and the triangles become congruent. So that in order that two triangles be congruent it is sufficient that some group of three conditions be satisfied. These sufficient conditions for congruence are none, other than S-A-S, A-S-A, S-S-S and A-A-S tests for congruence.

Exactly similar situation pertains in respect of similarity of two triangles. Out of the conditions necessary for similarity if some are satisfied then the rest automatically get satisfied. These are the sufficient conditions which ensure similarity. We will call such a group of sufficient conditions a test for similarity. It should be understood that these can be proved although we will not worry about proving them for the present. We will assume the truth about the tests for similarity.

§ 5.11 Tests for similarity:

(i) A-A-A test for similarity:

Given a one-to-one correspondence between vertices of two triangles if the corresponding angles are congruent then the two triangles are similar.

In other words, given a one-to-one correspondence between the vertices of two triangles if the corresponding angles are congruent then the corresponding sides are proportional.

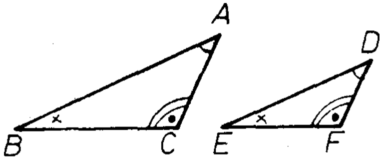


Fig. 5.43

Given: ABC $\longleftrightarrow$ DEF. If in $\triangle$ ABC and $\triangle$ DEF, $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$, then,

$$\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)} \quad \text{and}$$

$\triangle ABC \sim \triangle DEF$.

(ii) S-A-S test for similarity:

Given a one-to-one correspondence between the vertices of two triangles, if two sides of one are proportional to corresponding sides of the other and the angles included by them are congruent then the two triangles are similar.

In other words, given a one-to-one correspondence between the vertices of two triangles if two sides of one are proportional to the corresponding sides of the other and the angles included by these sides are congruent then all the corresponding angles of the two triangles are congruent and the corresponding sides of the two triangles are proportional.

Given $\triangle ABC$, $\triangle DEF$ and $ABC \longleftrightarrow DEF$.

If $\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)}$ and $\angle B \cong \angle E$, then

$\angle A \cong \angle D$, $\angle C \cong \angle F$,

$\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)}$

and $\triangle ABC \sim \triangle DEF$.

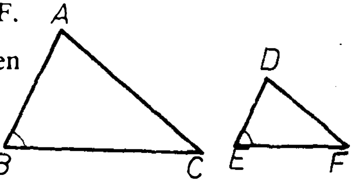


Fig. 5.44

(iii) S-S-S test for similarity:

Given a one-to-one correspondence between the vertices of two triangles. If the sides of one are proportional to the corresponding sides of the other then the two triangles are similar.

In other words, given a one-to-one correspondence between the vertices of two triangles. If the sides of one are proportional to the corresponding sides of the other then the corresponding angles of the two triangles are congruent.

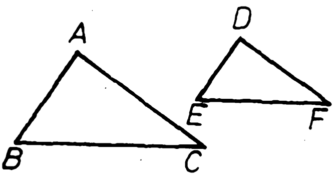


Fig. 5.45

Given $\triangle ABC$, $\triangle DEF$ and $ABC \longleftrightarrow DEF$.

If $\frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(EF)} = \frac{l(CA)}{l(FD)}$ then

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$ and $\triangle ABC \sim \triangle DEF$.

Exercise 5.5

1. In figures below angles with similar makrs are congruent. From the information given in the figure if you can conclude that triangles are similar, name the test that ensures similarity.

(i)

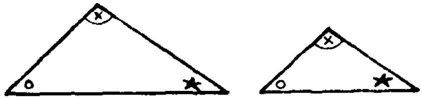


Fig. 5.46

(ii)

$\frac{l(LM)}{l(XY)} = \frac{l(LN)}{l(XZ)}$

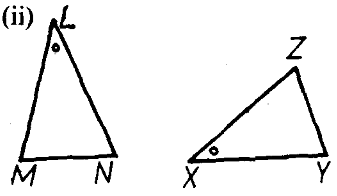


Fig. 5.47

(iii)

$$\frac{l(PQ)}{l(UV)} = \frac{l(QL)}{l(VW)} = \frac{l(LP)}{l(WU)}$$

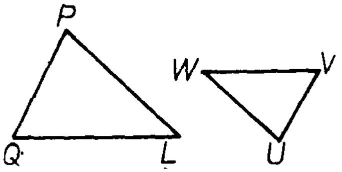
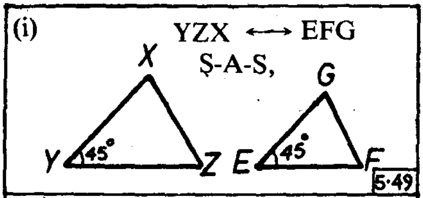


Fig. 5.48

2. In each figure below two triangles are given. In each figure some information alongwith a name of a test for similarity is given. If according to that test the two triangles are to be similar the information given in the figure is not enough. You have to supply additional information so that the test would work.



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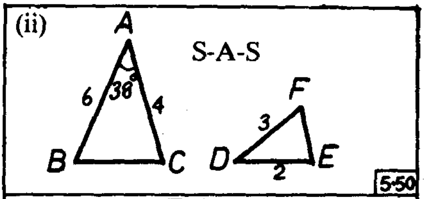

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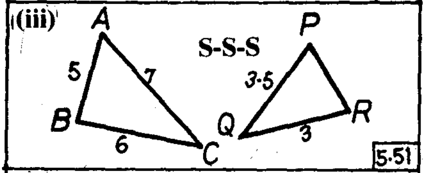

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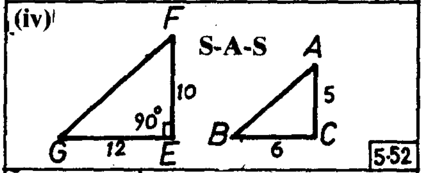

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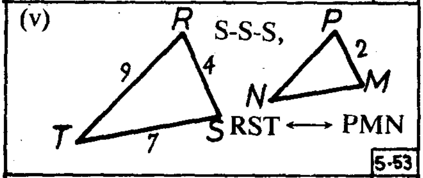

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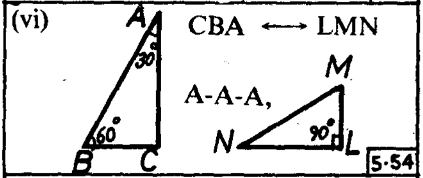

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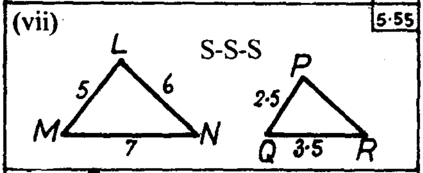

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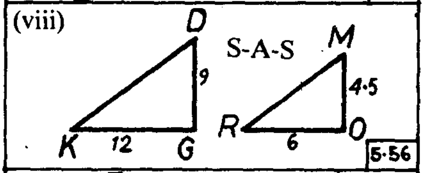

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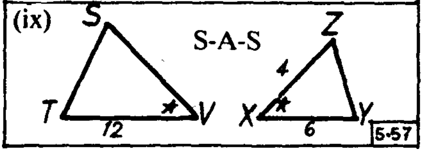

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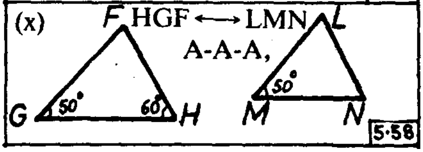

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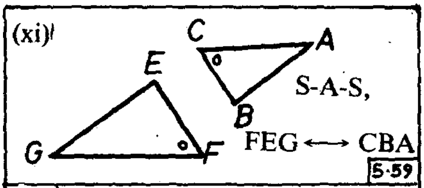

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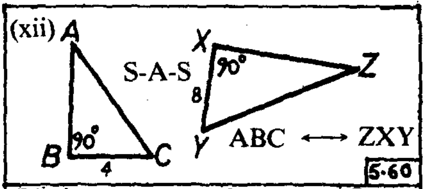

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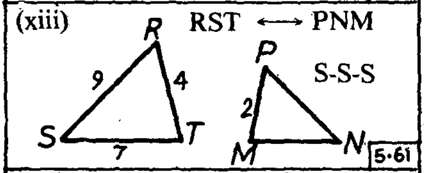

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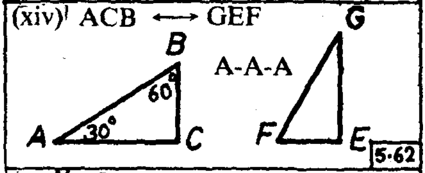

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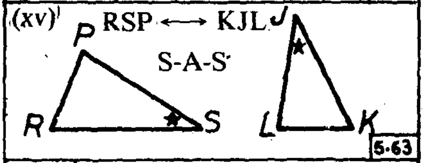

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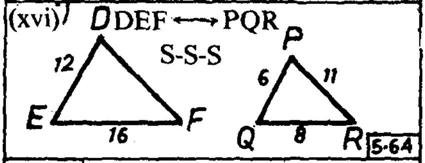

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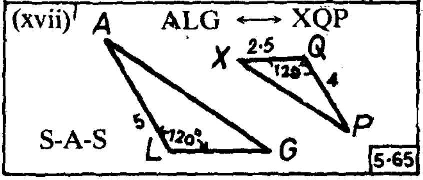

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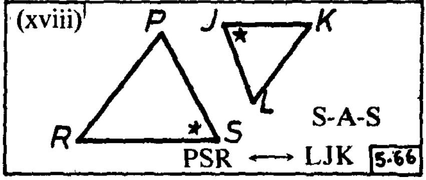


5·66

3. **(Theorem): If two triangles are similar the ratio of any two sides of one triangle is equal to the ratio of the corresponding sides of the other triangle.**

To prove this theorem we shall indicate the lengths of the sides of a triangle by small letters. For example, a means the length of the side opposite to the vertex A.

Given: $\triangle ABC \sim \triangle XYZ$

To prove:

$\frac{b}{a} = \frac{y}{x}$, $\frac{c}{b} = \frac{z}{y}$ and $\frac{a}{c} = \frac{x}{z}$

Proof: $\triangle ABC \sim \triangle XYZ$

$\therefore \frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

From $\frac{a}{x} = \frac{b}{y}$ we get,

$\frac{b}{a} = \frac{y}{x}$ and $\frac{a}{b} = \frac{x}{y}$

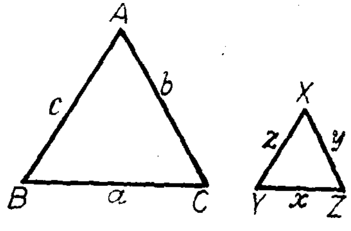


Fig. 5.67

From $\frac{b}{y} = \frac{c}{z}$ we get $\frac{c}{b} = \frac{z}{y}$ and $\frac{b}{c} = \frac{y}{z}$

From $\frac{a}{x} = \frac{c}{z}$ we get,

$\frac{a}{c} = \frac{x}{z}$ and $\frac{c}{a} = \frac{z}{x}$

Hence the ratio of any two sides of the first triangles is equal to the ratio of the corresponding sides in the second triangle.

Remember that this theorem will be very useful in the study of trignometry.

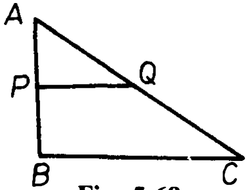


Fig. 5.68

4. In △ ABC, A-P-B, A-Q-C, seg PQ ∥ seg BC

Show: △ APQ ∼ △ ABC

5. In △ ABC seg AD ⊥ seg BC, seg EB ⊥ seg AC, B-D-C, A-E-C.
Show: △ ADC ∼ △ BEC

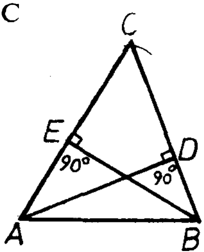


Fig. 5.69

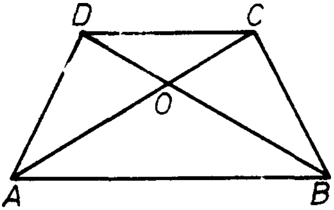


Fig. 5.70

6. In trapezium ABCD, seg DC ∥ seg AB Diagonals AC, BD intersect at O. Show that O divides seg DB and seg AC in the same ratio.

7. In fig. 5.71 △ ABC ∼ △ PQR. O and M are the incentres of the two triangles respectively. seg OD ⊥ seg BC, seg MN ⊥ seg QR.

Show that $\frac{l(OD)}{l(MN)} = \frac{l(BC)}{l(QR)}$.

We have, △ OBD ∼ △ MQN, △ ODC ∼ △ MNR.

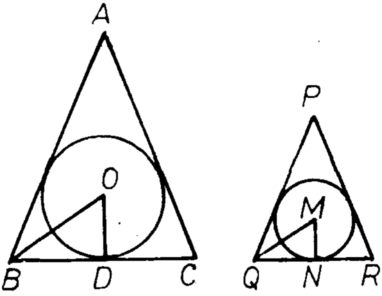


Fig. 5.71

$$\therefore \quad \frac{l(OD)}{l(MN)} = \frac{l(BD)}{l(QN)} = \frac{(l(DC)}{l(NR)} = \frac{l(BD) + l(DC)}{l(QN) + (NR)}$$

$$\therefore \quad \frac{l(OD)}{l(MN)} = \frac{l(BC)}{l(QR)}$$

[**The radii of incircles of two similar triangles are proportional to their corresponding sides.**]

8. In Fig. 5.72 △ ABC ~ △ PQR. set BL, seg QM are bisectors of ∠ ABC and ∠ PQR respectively. A-L-C, P-M-R.

Show: $\frac{l(BM)}{l(QM)} = \frac{l(AB)}{l(PQ)}$.

First show that [△ ABL ~ △ PQM]

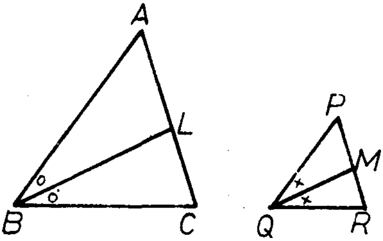


Fig. 5.72

[**In similar triangles corresponding angle bisectors are proportional to their corresponding sides.**]

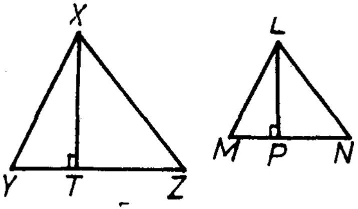


Fig. 5.73

9. In fig. 5.73, △ XYZ ~ △ LMN. seg XT ⊥ seg YZ, set LP ⊥ seg MN. Y-T-Z, M-P-N.

Show: $\frac{l(XT)}{l(LP)} = \frac{l(XY)}{l(LM)}$

(First show that △ XYT ~ △ LMP)

[**Corresponding heights of two similar triangles are proportional to the corresponding sides of the triangles.**]

10. In fig. 5.74, △ ABC ~ △ DEF. seg AX is the median of △ ABC. seg DR is the median of △ DEF.

Show: $\frac{l(AX)}{l(DR)} = \frac{l(AB)}{l(DE)}$.

(Show that △ ABX ~ △ DER.)

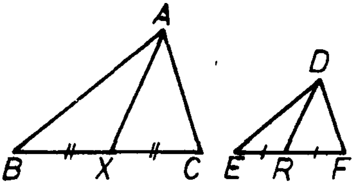


Fig. 5.74

[**Corresponding medians of two similar triangles are proportional to the corresponding sides of the triangles.**]

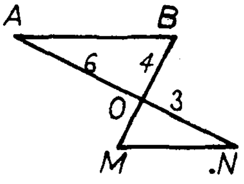


Fig. 5.75

11. In fig. 5.75 seg AB || seg MN.
From the information given in the figure find $l(\text{OM})$

12. (Theorem): Areas of two similar triangles are proportional to the squares of the lengths of their corresponding sides.

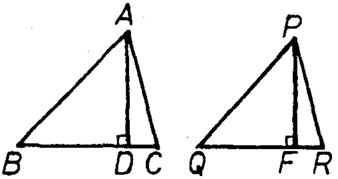


Fig. 5.76

Given: $\triangle ABC \sim \triangle PQR$. Seg AD and seg PF are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively.

To prove: $\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{[l(BC)]^2}{[l(QR)]^2}$

Proof: $\triangle ABC \sim \triangle PQR$

$\therefore \angle ABD \cong \angle PQF$ ————————(1)

and $\dfrac{l(AB)}{l(PQ)} = \dfrac{l(BC)}{l(QR)} = \dfrac{l(AC)}{l(PR)}$ ————————(2)

Now, since $\triangle$ ABD and $\triangle$ PQF are right angled triangles, and from (1) $\angle B \cong \angle Q$,

$\therefore \triangle ABD \sim \triangle PQF$

$\therefore \dfrac{l(AB)}{l(PQ)} = \dfrac{l(AD)}{l(PF)}$ ————————(3)

$\therefore \dfrac{l(AB)}{l(PQ)} = \dfrac{l(AD)}{l(PF)} = \dfrac{l(BC)}{l(QR)}$ ————————(4)

$$\text{Now, } \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{l(BC) \times l(AD)}{l(QR) \times l(PF)}$$

$$= \frac{l(BC)}{l(QR)} \times \frac{l(AD)}{l(PF)}$$

$$= \frac{l(BC)}{l(QR)} \times \frac{l(BC)}{l(QR)} \quad \text{[from (3) and (4)]}$$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{[l(BC)]^2}{[l(QR)]^2}$$

§ 5.12 Similarity of right angled triangles:

By A-A-A test two triangles become similar when their corresponding angles are congruent. In any triangle the sum of the measures of its angles is 180°. Because of this if two pairs of corresponding angles of two given triangles are congruent pairs then remaining pair is also a congruent pair and the triangles become similar.

In two right angled triangles, however if an acute angle of one triangle is congruent to an acute angle of the other then the two triangles become similar.

If a perpendicular is dropped on the hypotenuse of a right angled triangle from the opposite vertex we have two more right angled triangles. These two triangles along with the original triangle are similar to each other. This we now prove.

Theorem: The two right angled triangles obtained by dropping a perpendicular to the hypotenuse from the opposite vertex of a right angled triangle are similar to each other and also to the original triangle.

Given: In Δ ABC, $m\angle B = 90°$ seg BD $\perp$ seg AC, A-D-C.

To prove: Δ DBA $\sim$ Δ DCB $\sim$ Δ BCA.

Proof: In right angled triangles, Δ DBA and Δ BCA,

$\angle A \cong \angle A$

$\therefore$ Δ DBA $\sim$ Δ BCA (by A-A-A test) ——(1)

Similarly in right angled triangles, Δ DCB and Δ BCA, $\angle C \cong \angle C$

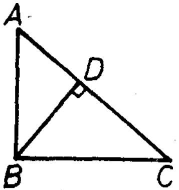


Fig. 5.77

$\therefore$ Δ DCB $\sim$ Δ BCA (by A-A-A test) ———(2)

$\therefore$ from (1) and (2),

Δ DBA $\sim$ Δ DCB $\sim$ Δ BCA.

Corollary: In a right angled triangle the perpendicular segment to the hypotenuse from the opposite vertex is the geometric mean of the segments into which the hypotenuse is devided.

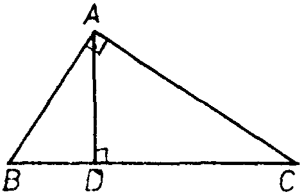


Fig. 5.78

Given: In Δ ABC, $m\angle A = 90°$, seg AD $\perp$ seg BC. B-D-C.

To prove: $[\,l(AD)\,]^2 = l(BD) \times (DC)$.

Proof: Δ ADB $\sim$ Δ CDA

$\therefore \dfrac{l(AD)}{l(DC)} = \dfrac{l(BD)}{l(AD)}$

$\therefore [\,l(AD)\,]^2 = l(BD) \times l(DC)$.

Exercise 5.6

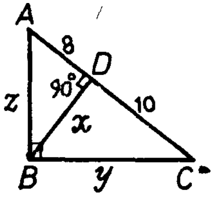


Fig. 5.79

1. In fig. 5.79, $m \angle B = 90°$, seg BD ⊥ seg AC, A-D-C, $l(AD) = 8$, $l(DC) = 10$, $l(BD) = x$, $l(BC) = y$, $l(AB) = z$. Find the values of x,y and z.
2. $\triangle ABC \sim \triangle PQR$ and if their areas are 81 sq cm and 144 sq cm respectively and $l(BC) = 3$ cm, find the length of seg (QR).
3. In two similar triangles the ratio of the corresponding sides is 2:3. If the area of the smaller triangle is 64 sq cm, find the area of the larger triangle.
4. In a trapezium ABCD, seg AB ∥ seg DC. $l(AB) = 3l(DC)$. The diagonals of the trapezium intersect at O.

   Show that $\frac{A(\triangle OCD)}{A(\triangle OAB)} = \frac{1}{9}$.

§ 5.14 Theorem of Pythagoras:

The square of the hypotenuse of a right angled triangle equals the sum of the squares of the other two sides.

We have already proved this theorem. At that time we looked at this theorem as a theorem concerned about certain areas. We know that by dropping a perpendicular on the hypotenuse of a right angled triangle from the opposite vertex we get three similar triangles. By using this fact we have a simpler proof for the theorem of Pythagoras.

To prove this theorem we shall indicate the lengths of the sides of a triangle by small letters a, b, c as shown in the figure.

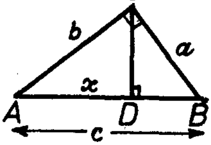


Fig. 5.80

Given : In $\triangle ABC$, $m \angle ACB = 90°$.
Seg CD ⊥ seg AB, $l(AD) = x$.

To prove: $[l(AB)]^2 = [l(AC)]^2 + [l(BC)]^2$
i.e. $c^2 = b^2 + a^2$

Proof: $\triangle ABC \sim \triangle ACD$. (similarity of right angled triangles)

$\therefore \frac{b}{x} = \frac{c}{b}$

$\therefore b^2 = cx$ ——————————(1)

Similarly $\triangle ABC \sim \triangle CBD$ (similarity of right angled triangles)

$\therefore \frac{a}{c-x} = \frac{c}{a}$

$\therefore a^2 = c(c-x) = c^2 - cx$ ————————(2)

$\therefore$ from (1) and (2),

$a^2 = c^2 - b^2$

$\therefore c^2 = a^2 + b^2$

$\therefore [l(AB)]^2 = [l(BC)]^2 + [l(AC)]^2$

Exercise 5.7

1. From the lengths of sides given in the following triangles determine whether they are right angled. Small letters denote lengths of sides opposite to vertices having corresponding capital lettes.

   (i) In $\triangle$ ABC, $a = 7, b = 24, c = 25$

   (ii) In $\triangle$ DEF, $d = 13, e = 12, f = 5$

   (iii) In $\triangle$ XYZ, $x = 5, y = 8, z = 10$

2. In $\triangle$ ABC, $m\angle$ ACB = 90°.
   P and Q are mid-points of seg AC and seg CB respectively.

   Show:

   (i) $4[l(AQ)]^2 = 4[l(AC)]^2 + [l(BC)]^2$

   (ii) $4[l(BP)]^2 = 4[l(BC)]^2 + [l(AC)]^2$

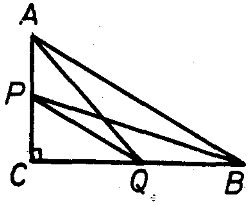


Fig. 5.81

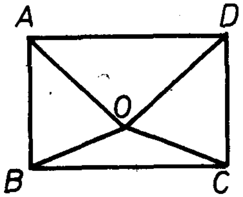


Fig. 5.82

3. Point O is inside a rectangle ABCD

   Show: $[l(OB)]^2 + [l(OD)]^2 = [l(OA)]^2 + [l(OC)]^2$

   (**Hint:** From O draw parallel segments to opposite sides of the rectangle and terminating on them. Use Pythagoras theorem.)

4. O is a point in the interior of $\triangle$ ABC. seg OD, seg OE and seg OF are pependiculars to seg BC, seg AC and seg AB respectively.

Show: $[l(AF)]^2 + [l(BD)]^2 + [l(CE)]^2 = [l(AE)]^2 + [l(CD)]^2 + [l(BF)]^2$

(**Hint:** Join O to A,B,C and use Pythagoras's theorem.)

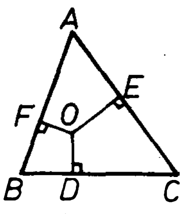


Fig. 5.83

§ 5.15 Theorem:

If a point O is a common point of either internal or external division of two chords AB and CD of a circle then

$$l(OA) \times l(OB) = l(OC) \times l(OD)$$

Case 1: O is a common point of internal division of the two chords AB, CD.

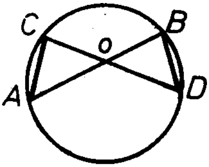


Fig. 5.84

Given: chord AB and chord CD of a circle intersect each other at O. The point O is in the interior of the circle.

To prove: $l(OA) \times l(OB) = l(OC) \times l(OD)$.

Proof: In $\triangle$ AOC and $\triangle$ DOB,

$$\left.\begin{aligned} \angle AOC &\cong \angle DOB \\ \text{and } \angle CAO &\cong \angle BDO \end{aligned}\right\} \text{(Give reasons)}$$

$$\therefore \triangle AOC \sim \triangle DOB$$

$$\therefore \frac{l(OA)}{l(OD)} = \frac{l(OC)}{l(OB)}$$

$$l(OA) \times l(OB) = l(OC) \times l(OD)$$

Case 2: O is a common point of external division of two chords AB and CD of a circle.

Given: chord AB and chord CD of a circle intersect each other at O. The point O is in the exterior of the circle.

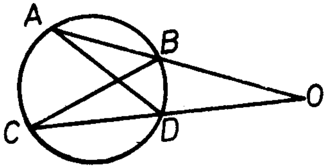


Fig. 5.85

To prove: $l(OA) \times l(OB) = l(OC) \times l(OD)$.

Proof: Draw seg AD and seg BC

In $\triangle$ ADO and $\triangle$ CBO

$\angle AOD \cong \angle COB$

$\angle DAO \cong \angle BCO$

$\therefore \triangle ADO \sim \triangle CBO$

$$\therefore \frac{l(OA)}{l(OC)} = \frac{l(OD)}{l(OB)}$$

$$\therefore l(OA) \times l(OB) = l(OC) \times l(OD)$$

§ 5.16 Theorem:

Given a point O outside a circle, a segment OT tangent to the circle at T and a secant from O intersecting the circle in A and B

then $[l(OT)]^2 = l(OA) \times l(OB)$

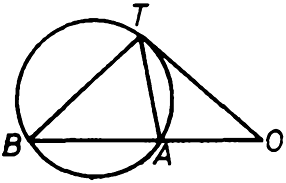


Fig. 5.86

Given: A point O outside a circle, seg OT is a tangent segment at T and secant through O intersects the circle at A and B.

To Prove: $[l(OT)]^2 = l(OA) \times l(OB)$

Construction: Join seg AT and seg BT

Proof: In $\triangle OTA$ and $\triangle OBT$ in the correspondence OTA ⟷ OBT,

$\angle TOA \cong \angle TOB$

$\angle ATO \cong \angle TBO$ (Give reason)

$\therefore \triangle OTA \sim \triangle OBT$

$$\therefore \frac{l(OT)}{l(OB)} = \frac{l(OA)}{l(OT)} \quad \therefore [l(OT)]^2 = l(OA) \times l(OB)$$

Using the above theorem we can easily prove the second case of the last theorem.

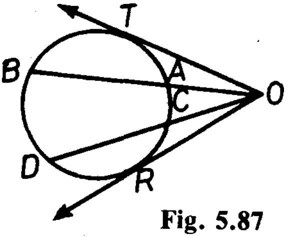


Fig. 5.87

In fig. 5.87,

$l(OA) \times l(OB) = [l(OT)]^2$

$l(OC) \times l(OD) = [l(OT)]^2$

$\therefore l(OA) \times l(OB) = l(OC) \times l(OD)$

We also know that two tangent segments can be drawn to a circle from a point outside the circle and the segments are congruent. We verify this property using the above theorem.

In fig. 5.87, $[l(OT)]^2 = l(OA) \times l(OB)$

$[l(OR)]^2 = l(OA) \times l(OB)$

$\therefore [l(OT)]^2 = [l(OR)]^2 \quad \therefore l(OT) = l(OR).$

Exercise 5.8

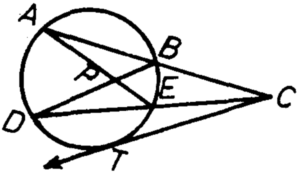


Fig. 5.88

1. In fig. 5.88 if
   (i) $l(AB) = 8$, $l(BC) = 6$, $l(CE) = 7$, then $l(DC) = ?$
   (ii) $l(CT) = 6$, $l(DE) = 5$, then $l(CE) = ?$
   (iii) $l(AP) = 5$, $l(EP) = 4$, $l(PB) = 2$ then $l(DP) = ?$
   (iv) $l(DP) = 3$, $l(DB) = 5$, $l(PE) = 4$, then $l(AP) = ?$

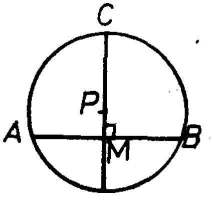


Fig. 5.89

2. In fig. 5.89, diameter CD $\perp$ chord AB of a circle of radius r and centre P. The point of intersection of seg CD and seg AB is M. If,
   (i) $l(PM) = 8$, $r = 10$, find $l(AB)$
   (ii) $l(AB) = 12$, $r = 20$, find $l(PM)$

3. seg MN is a diameter of a circle of radius r. seg PM is tangent to the circle at M. seg PN intersects the circle at Q.
   Show: $l(PN) \times l(QN) = 4r^2$
   (Hint: $[\,l(PN)\,]^2 = 4r^2 + [\,l(PM)\,]^2$,
   $[\,l(PM)\,]^2 = l(PQ) \times l(PN)$)

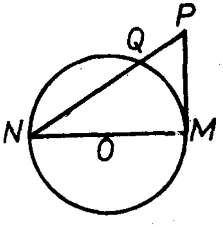


Fig. 5.90

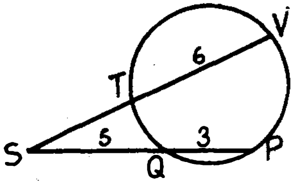


Fig. 5.91

4. In fig. 5.91, $l(SQ) =$, $l(PQ) = 3$, $l(VT) = 6$, $l(ST) = m$, find m.

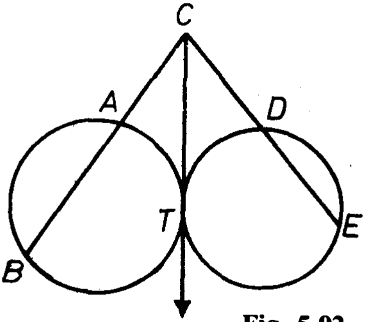


Fig. 5.92

5. In fig. 5.92, seg CT is a common tangent segment to two circles which touch each other externally at T. Secants from C intersect one circle in A and B and the other in D and E.
   Show: $l(CA) \times l(CB) = l(CD) \times l(CE)$.

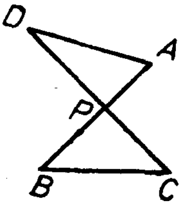


Fig. 5.93

6. In fig. 5.93 seg AB and seg CD intersect at P and $l(PA) \times l(PB) = l(PC) \times l(PD)$.
   Show: (i) $\triangle PBC \sim \triangle PDA$
   (ii) to what angle is $\angle$ DAP congruent ?
   (iii) Show that points A, B, C and D are concyclic.

7. In $\triangle$ ABC perpendicular segments, seg AD, seg BE and seg CF, from the vertices of the triangle to the opposite sides intersect at a point M. Show that a circle passes through the points A, B, D and E.

$l(MA) \times l(MD) = l(MB) \times (---)$

(Fill in the blank properly).

Similarly,

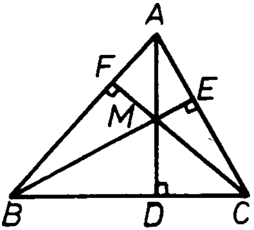


Fig. 5.94

$l(AM) \times l(MD) = l(BM) \times l(---)$

$= l(CM) \times l(---)$

(Fill in the blanks properly).

Miscellaneous Exercise — 5

1. State whether the following statements are true or false. Give a counter example to establish that it is false.
   - (i) Two triangles of equal heights are equal in areas.
   - (ii) If two triangles are similar they are congruent.
   - (iii) If two triangles are congruent they are similar.
   - (iv) Two right angled triangles are always similar.
   - (v) In two right angled triangles if the ratio of their bases is equal to the ratio of their altitudes then they are similar.
   - (vi) $\triangle ABC \sim \triangle PQR$ and $\triangle ABC \sim LMN$,

     then, $\frac{l(AB)}{l(PQ)} = \frac{l(AB)}{l(LM)}$.

2. Fill in the blanks properly in the following statements.
   - (i) If a line divides the two sides of a triangle proportionally then it is ____________ to the third side.
   - (ii) If two segments divide the two given sides of a triangle proportionally then they are ____________ to each other.
   - (iii) In a $\triangle$ ABC, P is a point on seg AB such that $l(AP) = \frac{1}{3} l(AB)$.

     Then $A(\triangle APC) =$ ____________ $A(\triangle ABC)$
   - (iv) In two similar triangles if the ratio of the corresponding sides is $\frac{8}{5}$ then the ratio of the corresponding altitudes is ____________.
   - (v) In two similar triangles if the ratio of areas is $\frac{9}{4}$ then the ratio of the corresponding sides is ____________.

)

3. In two similar triangles the ratio of the corresponding sides is $\frac{3}{4}$. Fill in the following blanks with suitable values for:—
   (i) ratio of their corresponding altitudes ____________
   (ii) ratio of their corresponding medians ____________
   (iii) ratio of their corresponding angle bisectors ____________
   (iv) ratio of radii of their incircles _ _ _ _ _ _ _
   (v) ratio of their areas ____________

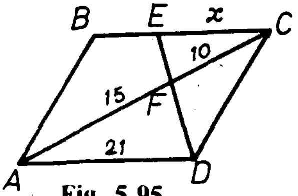


Fig. 5.95

4. In a parallelogram ABCD, E is a point on seg BC. If F is the point of intersection of seg ED and diagonal AC, find the value of x from the information given in the figure.

5. In a trapezium ABCD, seg AD || seg BC and E is the point of intersection of diagonals AC and BD. Find the value of a x from the information given in the figure.

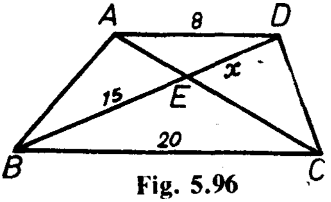


Fig. 5.96

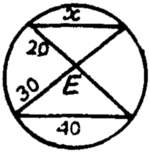


Fig. 5.97

6. Find the value of x from the information given in the figure.

7. In a trapezium ABCD seg AB || seg DC. Points E and F are the points on seg AB and seg DC respectively. If G is the point of intersection of the diagonal DB and seg EF, show that $l(GB) \times l(DF) = l(GD) \times l(EB)$.

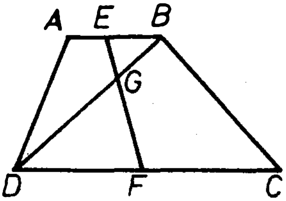
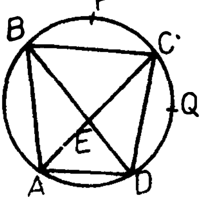


Fig. 5.98

8. In a triangle ABC, seg BD ⊥ seg AC and A-D-C; seg DE ⊥ seg AB and B-E-A. Then prove that $l(DE) \times l(AD) = l(BD) \times l(AE)$.



9. If fig. 5.99 m(arc BPC) = m(arc CQD). Then show that $l(AC) \times l(EC) = [\,l(BC)\,]^2$

Fig. 5.99

10. In $\triangle PQR$, $m\angle PRQ = 90°$, seg $PS \perp$ seg PQ. Then from the information given in the figure answer the following questions.

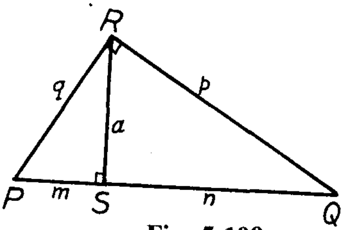


Fig. 5.100

(i) If $m = 27$, $n = 3$, then find the values of a, p and q.
(ii) If $m = 24$, $n = 6$, then find the values of a, p and q.
(iii) If $p = 15$, $n = 9$, then find the values of m and q.
(iv) If $a = 8$, $m = 16$, then find the values of p and q.

11. The areas of two similar triangles are 16 and 25 sq units. Find the ratio of the areas of their incircles.

12. In two similar triangles the area of the larger triangle is 9 times the area of the smaller triangle. If one side of the smaller triangle is 5 cm. long find the length of the corresponding side of the larger triangle.

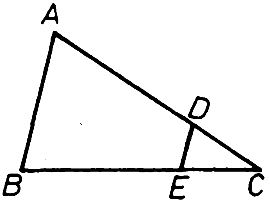


Fig. 5.101

13. In a $\triangle ABC$, point D is the point of seg AC such that $l(AD) = 2l(DC)$ and point E is the point on seg BC such that seg DE $\parallel$ seg AB. If $A(\square ABED) = 40$ sq. units then find the area of $\triangle ABC$.

14. In two similar triangles the sides of the smaller triangle are 5, 7 and 8. If the perimeter of the larger triangle is 100 find the sides of the larger triangle.

15. Solve the following problems from the information given in the adjoining figures.

(a) In fig. 5.102 if

(i) $l(AE) = 10$, $l(EB) = 6$, $l(CE) = 12$, then find $l(ED)$.
(ii) $l(AB) = 15$, $l(EB) = 8$, $l(ED) = 4$, then find $l(CE)$.
(iii) $l(AE) = 6$, $l(ED) = 4$, $l(CD) = 13$, then find $l(EB)$.

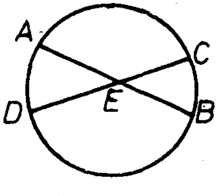


Fig. 5.102

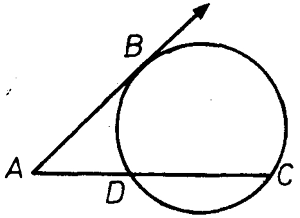


Fig. 5.103

(b) In fig. 5.103 if
- (i) $l(AC) = 16$, $l(AD) = 4$, then $l(AB) = ?$
- (ii) $l(AB) = 6$, $l(AD) = 3$, then $l(AC) = ?$
- (iii) $l(AC) = 20$, $l(AB) = 10$, then $l(AD) = ?$

(c) In fig. 5.104 if
- (i) $l(AD) = 6$, $l(OD) = 9$, then $l(AB) = ?$
- (ii) $l(AD) = 5$, $l(AB) = 10$, then $l(OD) = ?$
- (iii) $l(AB) = 12$, $l(AC) = 18$, then $l(OD) = ?$

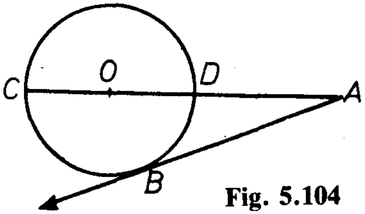


Fig. 5.104

□ □ □

Chapter 6

TRIGONOMETRY

§ **6.1** We make use of a scale or a tape if we wish to determine the length, breadth and height of a table. By physically measuring we may get to know how long and how wide a farm is. But certain heights and distances are such that it may not be possible to measure them physically. How can we determine the actual height of a kite floating steadily but high above ir the sky ? If we decide to measure the height of a large tower or the width of a big river there are many hurdles that cross our way.

But it is not so difficult as may appear on the surface to find a solution to these problems.

Please study the example solved below carefully. The principle which ensures solution of the problem below also helps us to answer problems posed above.

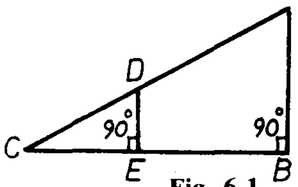


Fig. 6.1

A tower has base B and top A. C is a point on the ground at a distance of 30 m from B. A staff 1 m long is made to stand vertically at a point E between C and B such that its other end D appears to be in line with C and A. Distance of E from C is 1.5 m.

From this information it is now possible to find the height of the tower without actually measuring it.

In right angled triangles △ DEC, △ ABC

$\angle DCE \cong \angle ACB$

$\therefore \triangle DCE \sim \triangle ACB$

$$\therefore \frac{l(AB)}{l(DE)} = \frac{l(BC)}{l(CE)}$$

$$\therefore \frac{l(AB)}{1} = \frac{30}{1.5}$$

$\therefore l(AB) = 20$ m.

Please note that we have made use of similarity of right angled triangles to solve the problem.

§ **6.2 Trigonometry:** Literally the word trigonometry means measurements of a triangle. The basis on which the structure of this branch of mathematics is built is similarity of triangles. Knowledge of trigonometry is needed in the study of physics, Geography, astronomy, naval and many other sciences.

Since properties of similar triangles are necessary for proper understanding of trigonometry let us revise some important points concerning similarity of triangles by working out some problems.

§ **6.3** In exercises given below some information is provided about the sides and angles of two triangles. Congruent angles are shown by identical marks. On the basis of the information given fill in the blanks properly.

(1) △ ABC ∼ △ DEF

$x =$ ____

$y =$ ____

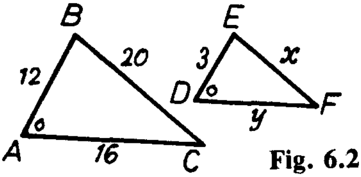
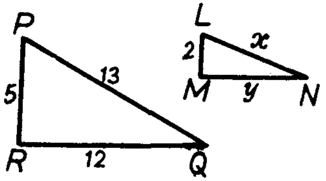


Fig. 6.2

(2)



△ PQR ∼ △ LNM

$x =$ ______

$y =$ ______

Fig. 6.3

(3)

△ XYZ ∼ △ STR

$$\frac{a}{\square} = \frac{b}{\square} = \frac{8}{\square}$$

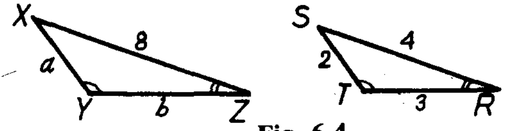
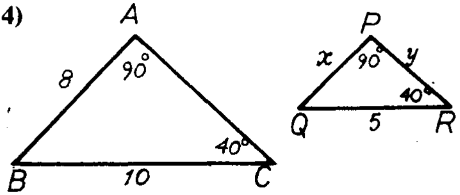


Fig. 6.4

(4)



$\frac{l(PR)}{l(AC)} = \_\_\_\_ = \_\_\_$

$x = \_\_\_\_\_$

$y = \_\_\_\_\_$

Fig. 6.5

(5)

Δ ADE $\sim$ Δ ABC

$x = 15, s = 5$

$t = 3, y = \_\_\_$

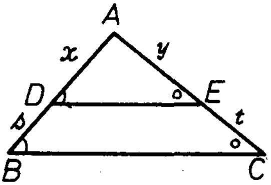
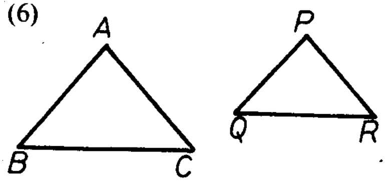


Fig. 6.6

(6)



Δ ABC $\sim$ Δ PQR

Fig. 6.7

$\frac{l(AB)}{l(AC)} = \frac{l(PQ)}{-}$, $\frac{l(BC)}{l(AC)} = \frac{-}{l(PR)}$, $\frac{l(BC)}{l(AB)} = \frac{(QR)}{-}$

§ 6.4 Right angle triangles and similarity:

If two triangles are similar, ratio of the lengths of any two sides of the first triangle is equal to the ratio of the lengths of the corresponding sides of the other. This property has extensive use in the study of trigonometry. We will now see how certian ratios can be associated with every acute angle. While doing this we will keep in mind that for right angled triangles, however, it is enough, if one acute angle of the one triangle is congruent to an acute angle of the other, to make these triangles similar.

§ 6.5 Acute angle and some ratios:

In the figure the triangles shown i.e. ΔOP_1Q_1, ΔOP_2Q_2 have a common angle at O. Let the common angle be denoted by θ.

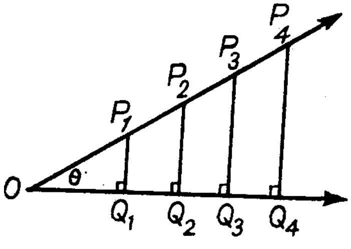


Fig. 6.8

$$\therefore \ \Delta OP_1Q_1 \sim \Delta OP_2Q_2$$

$$\therefore \ \frac{l(P_1Q_1)}{l(OP_1)} = \frac{l(P_2Q_2)}{l(OP_2)} \qquad (1)$$

$$\frac{l(OQ_1)}{l(OP_1)} = \frac{l(OQ_2)}{l(OP_2)} \qquad (2)$$

$$\frac{l(P_1Q_1)}{l(OQ_1)} = \frac{l(P_2Q_2)}{l(OQ_2)} \qquad (3)$$

From the above fact it can be seen easily that,

$$\Delta OP_1Q_1 \sim \Delta OP_2Q_2 \sim \Delta OP_3Q_3 \sim \Delta OP_4Q_4 \ldots\ldots$$

$$\therefore (1) \quad \frac{l(P_1Q_1)}{l(OP_1)} = \frac{l(P_2Q_2)}{l(OP_2)} = \frac{l(P_3Q_3)}{l(OP_3)} = \frac{l(P_4Q_4)}{l(OP_4)} = \ldots\ldots$$

$$(2) \quad \frac{l(OQ_1)}{l(OP_1)} = \frac{l(OQ_2)}{l(OP_2)} = \frac{l(OQ_3)}{l(OP_3)} = \frac{l(OQ_4)}{l(OP_4)} = \ldots\ldots$$

$$(3) \quad \frac{l(P_1Q_1)}{l(OQ_1)} = \frac{l(P_2Q_2)}{l(OQ_2)} = \frac{l(P_3Q_3)}{l(OQ_3)} = \frac{l(P_4Q_4)}{l(OQ_4)} = \ldots\ldots$$

In respective triangles,

seg P_1Q_1, seg P_2Q_2, seg P_3Q_3... are sides of opposite to θ.

seg OQ_1, seg OQ_2, seg OQ_3... are sides adjacent to θ.

seg OP_1, seg OP_2, seg OP_3... are hypotenuses.

From the above discussion it is clear that if a given acute angle is associated with any right angled triangle which has this as one of its angles then the values of the following ratios remain invariant i.e. do not change.

(1) $\dfrac{\text{Length of the opposite side of the angle}}{\text{Length of the Hypotenuse}}$

(2) $\dfrac{\text{Length of the adjacent side}}{\text{Length of the Hypotenuse}}$

(3) $\dfrac{\text{Length of the opposite side}}{\text{Length of the adjacent side}}$

These ratios are called **trigonometrical ratios.**

§ 6.6 Sine ratio of an acute angle:

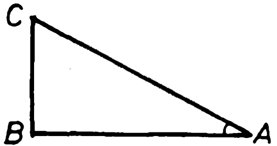


Fig. 6.9

If an acute ∠ BAC is included by a triangle Δ ABC in which $m \angle B = 90°$, then the ratio $\frac{l(BC)}{l(CA)}$ is called sine ratio of the ∠ BAC.

We write sin ∠ BAC to indicate the sine ratio of ∠ BAC. It is read as sine angle BAC,

$$\sin \angle BAC = \frac{l(BC)}{l(CA)}$$

When no cause for confusion arises in place of three letter name we use one letter name of the angle.

sin ∠ A means the sine ratio of the angle A. Some times we simply write sin A in place of sin ∠ A.

In this notation the letter A itself implies that ∠ A is meant. Sometimes in place of the name of the angle its measure is written. sin θ means the sine of the angle whose measure is θ °. sine 36 or sin 36° means the sine of the angle whose measure is 36°

We can also consider the sine of ∠ ACB in △ BCA [see fig. 6.9].

$$\sin \angle ACB = \frac{l(AB)}{l(AC)} \quad \text{or} \quad \sin C = \frac{l(AB)}{l(AC)}$$

From the discussion above given any angle and a right angled triangle that includes it,

$$\frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$$

is known as the **sine ratio** of the angle.

§ 6.7 In examples below some right angled triangles are shown, with their help, fill in properly the blanks shown.

(1)

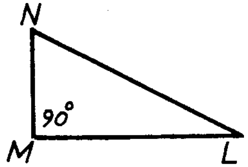


Fig. 6.10

sin ∠ NLM = ________

sin ∠ LNM = ________

(2)

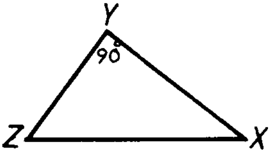
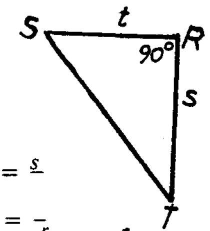


Fig. 6.11

sin X = ____

sin Z = ____

(3)



sin S = $\frac{s}{\ }$

sin T = $\frac{\ }{r}$

Fig. 6.12

(4)

sin 30 = ___

sin 60 = ___

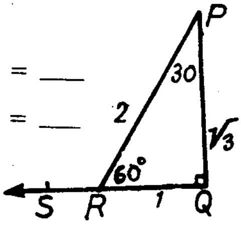


Fig. 6.13

(5)

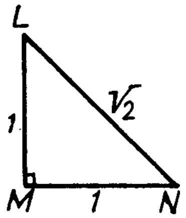


Fig. 6.14

sin 45 = ______

(6)

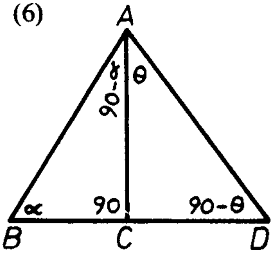


Fig. 6.15

Sin α = _ _ _ , Sin θ = _ _ _

Sin $(90-\alpha)$ = _ _ _ , Sin $(90-\theta)$ = _ _

§ 6.8 Cosine ratio associated with an acute angle:

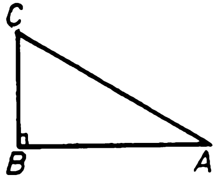


Fig. 6.16

If an acute ∠ BAC is included by the △ABC in which $m\angle B = 90°$, then $\frac{l(BA)}{l(AC)}$ is called the cosine ratio of ∠ BAC.

cos ∠ BAC means the cosine ratio of ∠ BAC, cos ∠ BAC is read as cosine of angle BAC or cos of anlge BAC. If one letter name is used for the angle in the above figure, cos ∠ A or simply cos A, will mean cos ∠ BAC. cos 49 or cos 49° means cosine of the angle whose measure is 49°.

We can also consider cosine ratio of ∠ ACB in the above △ ABC.

$$\cos\angle ACB = \frac{l(BC)}{l(AC)} \text{ or } \cos C = \frac{l(BC)}{l(AC)}.$$

From the discussion above it is clear that if an acute angle and right angled triangle that includes it, is given, then the ratio,

$$\frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$$

is known as **cosine ratio** of the given angle.

§ 6.9 In examples below are shown right angled triangles. Using information given in the figures fill in properly the blanks shown.

1.

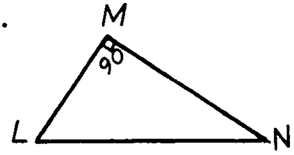


Fig. 6.17

cos ∠ NLM = ______

cos ∠ LNM = ______

2.

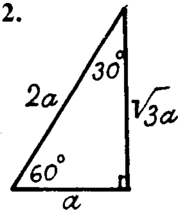


Fig. 6.18

sin 30 = ______

sin 60 = ______

cos 30 = ______

cos 60 = ______

3.

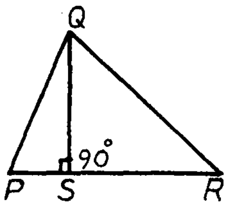


Fig. 6.19

cos ∠ PQS = ______
cos ∠ RQS = ______
cos ∠ SPQ = ______
cos ∠ SRQ = ______

4.

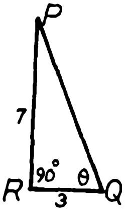


Fig. 6.20

l(PQ) = ______
sin θ = ______
cos θ = ______

5.

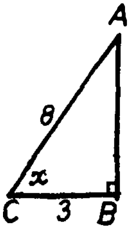


Fig. 6.21

l(AB) = ______
cos x = ______
sin x = ______

6.

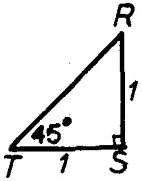


Fig. 6.22

l(RT) = ______
cos 45 = ______
sin 45 = ______

7.

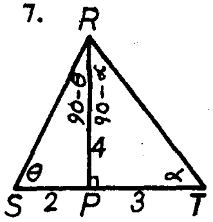


Fig. 6.23

l(SR) = ______, l(RT) = ______
sin(90 − α) = ______
cos(90 − α) = ______
sin θ = ______
cos θ = ______
sin(90 − θ) = ______
cos(90 − θ) = ______

§ 6.10 Tangent ratio associated with an acute angle:

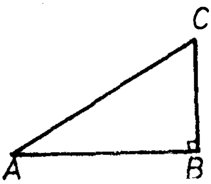


Fig. 6.24

If an acute ∠ BAC is included by a right angled △ ABC in which m ∠ B = 90°, then the ratio $\frac{l(BC)}{l(BA)}$ is known as the tangent ratio of ∠ BAC.

tan ∠ BAC means tangent ratio of ∠ BAC. This is read as tangent or tan angle BAC. If one letter name of an angle is used, tan ∠ A or tan A means tan ∠ BAC, tan 45 or tan 45° means tangent of angle whose measure is 45°.

$$\tan \angle BAC = \frac{l(BC)}{l(AB)} \text{ or } \tan A = \frac{l(BC)}{l(AB)}$$

We can also consider the tangent ratio of ∠ ACB in △ ABC.

$$\tan \angle ACB = \frac{l(AB)}{l(BC)} \text{ or } \tan C = \frac{l(AB)}{l(BC)}$$

From the discussion above it is clear that if an acute angle and a right angled triangle that includes it, is given, then,

$$\frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

is known as the **tangent ratio** of that angle.

§ **6.11** From information given in adjoining figures fill in properly the blanks shown.

1.

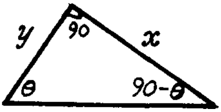


Fig. 6.25

$\tan \theta$ = _______

$\tan(90 - \theta)$ = ______

2.

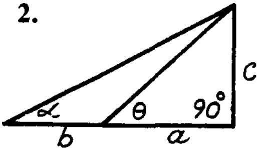


Fig. 6.26

$\tan \alpha$ = _______

$\tan \theta$ = _______

3.

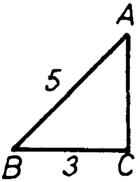


Fig. 6.27

$l(AC)$ = _______

$\tan \angle ABC$ = ___

$\tan \angle CAB$ = ___

§ 6.12 Relation between sine, cosine and tangent ratios of an acute angle:

From the figure 6.28,

$$\sin \theta = \frac{b}{c}, \cos \theta = \frac{a}{c}, \tan \theta = \frac{b}{a}$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{c} \times \frac{c}{a}$$

$$= \frac{b}{a}$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \tan \theta$$

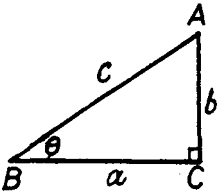


Fig. 6.28

Tangent ratio of an angle is the ratio of its sine and cosine ratios.

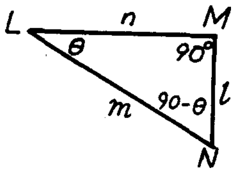


Fig. 6.29

§ 6.13 Relation between trigonometrical ratios of two complimentary angles:

In the figure 6.29 △ LMN is a right angled triangle.
$m\angle M = 90°$, ∠ L and ∠ N are complementary angles of each other.

If $m\angle L = \theta°$ then $m\angle N = (90 - \theta)°$

$\therefore \sin\theta = \frac{l}{m}$, $\cos(90 - \theta) = \frac{l}{m}$

$\therefore$ **$\sin\theta = \cos(90 - \theta)$**

Similarly $\cos\theta = \frac{n}{m}$, $\sin(90 - \theta) = \frac{n}{m}$

and $\therefore$ **$\cos\theta = \sin(90 - \theta)$**

$\tan\theta = \frac{l}{n}$ and $\tan(90 - \theta) = \frac{n}{l}$

$\therefore \tan\theta \times \tan(90 - \theta) = \frac{l}{n} \times \frac{n}{l} = 1$

$\therefore$ **$\tan\theta \times \tan(90 - \theta) = 1$**

(1) Sine and cosine ratios of an acute angle are respectively equal to cosine and sine ratios of the corresponding angle.

(2) The product of tangent ratios of angles complementary to each other is 1. or tangent ratios of complementary angles are reciprocals of each other. i.e.

$\tan x \times \tan(90 - x) = 1$

& 6.14 Fundamental formula of Trigonometry:

From the figure 6.30,

$\sin A = \frac{a}{b}$, $\cos A = \frac{c}{b}$

and $b^2 = a^2 + c^2$

$\therefore (\sin A)^2 = \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

$(\cos A)^2 = \left(\frac{c}{b}\right)^2 = \frac{c^2}{b^2}$

$$\therefore (\sin A)^2 + (\cos A)^2 = \frac{a^2}{b^2} + \frac{c^2}{b^2}$$
$$= \frac{a^2 + c^2}{b^2}$$
$$= \frac{b^2}{b^2}$$
$$= 1$$

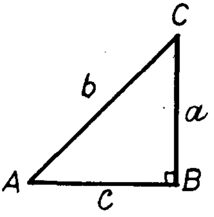


Fig. 6.30

Generally we write $\sin^2 A$, $\cos^2 A$ for $(\sin A)^2$ and $(\cos A)^2$ respectively.

$\therefore$ $\sin^2 A + \cos^2 A = 1$ (1)

Sum of squares of sine and cosine ratios of an acute angle is 1.

from (1)

$\sin^2 A = 1 - \cos^2 A$ (2)

$\cos^2 A = 1 - \sin^2 A$ (3)

§ **6.15** The three trigonometrical ratios associated with an acute angle are related to each other. If the value of one of them is known the value of other ratios could be determined. All the trigonometrical ratio are positive numbers. For a given acute angle these ratios never take a negative value.

§ 6.16 Solved Examples:

[Note - In all examples angles are acute angles]

(1) If $\sin A = \frac{3}{5}$ find the value of cos A.

Method 1

From the basic formula we have

$$\begin{aligned}\cos^2 A &= 1 - \sin^2 A\\ &= 1 - \left(\tfrac{3}{5}\right)^2\\ &= 1 - \tfrac{9}{25}\\ &= \tfrac{16}{25}\end{aligned}$$

$\therefore$ $\cos A = \frac{4}{5}$

Method 2

In the right angled triangle ABC , $m\angle B = 90°$ which includes $\angle A$, let the opposite side to $\angle A$ be 3, and hypotenuse be 5.

By the theorem of Pythagoras,

$[l(AB)]^2 = 25 - 9 = 16$

$\therefore$ $l(AB) = 4$

$\therefore$ $\cos A = \frac{4}{5}$

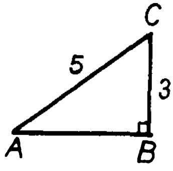


Fig. 6.31

(2) If $\cos \theta = \frac{12}{13}$ find the values of $\sin \theta$ and $\tan \theta$

Method 1

From the fundamental formula.

$$\sin^2\theta = 1 - \cos^2\theta$$
$$= 1 - \left(\tfrac{12}{13}\right)^2$$
$$= 1 - \tfrac{144}{169}$$
$$= \tfrac{25}{169}$$

$$\therefore \sin\theta = \tfrac{5}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Method 2

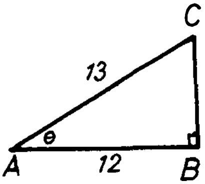


Fig. 6.32

In a right angled triangle ABC which includes angle θ, let $m\angle B = 90°$. $m\angle A = \theta$, $l(AB) = 12$, $l(AC) = 13$. Then by the theorem of Pythagoras,

$$[l(BC)]^2 = 169 - 144$$
$$= 25$$
$$\therefore l(BC) = 5$$
$$\therefore \sin\theta = \tfrac{5}{13} \text{ and } \tan\theta = \tfrac{5}{12}$$

3. If $\tan A = \frac{3}{4}$ find the values of sin A, cos A.

Method 1

$$\tan A = \frac{\sin A}{\cos A}$$
$$\therefore \frac{\sin A}{\cos A} = \frac{3}{4}$$
$$\therefore \sin A = \frac{3}{4}\cos A$$

But $\sin^2 A + \cos^2 A = 1$

$$\therefore \left(\tfrac{3}{4}\cos A\right)^2 + \cos^2 A = 1$$
$$\frac{9}{16}\cos^2 A + \cos^2 A = 1$$
$$\frac{25}{16}\cos^2 A = 1 \therefore \cos^2 A = \frac{16}{25}$$
$$\therefore \cos A = \frac{4}{5}$$
$$\sin A = \frac{3}{4}\cos A = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$

Method 2

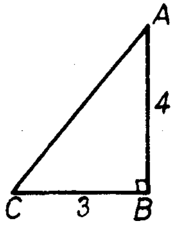


Fig. 6.33

In the figure 6.33,
let $m \angle B = 90°$, $l(BC) = 3$, $l(AB) = 4$
By Pythagoras theorem,
$[l(AC)]^2 = 4^2 + 3^2 = 16 + 9 = 25$
$\therefore\ l(AC) = 5$
$\therefore\ \sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$

(4) Show that $\sin^2 34 + \sin^2 56 = 1$.

Here $34 + 56 = 90$ and so the angles 34°, 56° are complementary angles.

$\therefore\ \sin 56 = \cos 34$

$$\therefore\ \sin^2 34 + \sin^2 56 = \sin^2 34 + \cos^2 34$$
$$= 1$$

(5) In the figure $m \angle B = 90°$, $l(AB) = 20$,
$\cos A = \frac{40}{41}$ find $l(AC)$

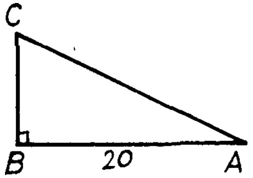


Fig. 6.34

$$\cos A = \frac{l(AB)}{l(AC)}$$

But $\cos A = \frac{40}{41}$ and $l(AB) = 20$.

$$\therefore\ \frac{20}{l(AC)} = \frac{40}{41}$$

$$\therefore\ l(AC) = \frac{41}{2} = 20.5$$

Excercise 6.1

1. $\sin A = \frac{4}{5}$, find $\cos A$.
2. $\cos A = \frac{40}{41}$, find $\sin A$, $\tan A$.
3. $\cos \theta = \frac{12}{13}$, then $\sin \theta = ?$; $\tan \theta = ?$
4. If $\tan \alpha = \frac{1}{2\sqrt{2}}$ then find $\sin \alpha$, $\cos \alpha$
5. If $\sin \theta = \frac{5}{7}$, then find the values of $\cos \theta$, $\tan \theta$.
6. $\tan B = \frac{1}{\sqrt{3}}$ find the values of $\sin B$ and $\cos B$.

7. If $\cos x = \frac{3}{4}$ then $\sin x = ?$, $\tan x = ?$

8. If $\sin A = 20 \cos A$, then find the values of $\sin A$, $\cos A$, and $\tan A$.

9. $\sin A = \frac{7}{25}$ find $\sin A + \cos A$ and $\sin A \cdot \cos A$.

10. $9 \sin A - 40 \cos A = 0$ find the value of $\sin A$, $\cos A$.

11. If $\sin \theta = \frac{60}{61}$, find $\cos \theta$ and $\tan \theta$.

(12) In Δ ABM $m \angle M = 90°$
$l(AB) = 10$, $\sin B = \frac{1}{2}$
Find $l(BM)$, $\cos B$ and $\tan B$.

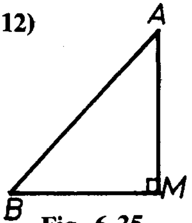


Fig. 6.35

13. A ladder rests against a wall. The upper end is 3 m from the ground. Lower end is at a distance of 2 m from the wall. Find the sine ratio of the angle made by the ladder with the ground.

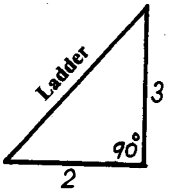


Fig. 6.36

(14) In the figure 6.37
$m \angle B = 90°$, $l(AB) = 12$,
$m \angle A = \theta$, $\cos \theta = \frac{4}{5}$
find $l(AC)$, $\sin \theta$ and $\cos \theta$.

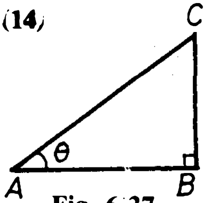


Fig. 6.37

(15) In the figure 6.38 $m \angle P = 90°$
$m \angle R = x$
$l(QR) = 25$, $\sin x = \frac{3}{5}$
Find $l(PQ)$, $l(PR)$ and $\cos x$, $\tan x$.

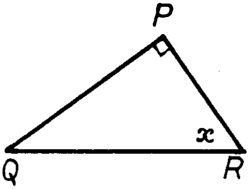


Fig. 6.38

§ 6.17 Trigonometrical ratios of angles of measures 30°, 60°, 45°.:

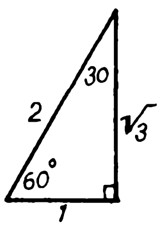


Fig. 6.39

In 30-60-90 triangle the side opposite to angle 30° is half in length of the hypotenuse. Using this fact we can determine the trigonometrical ratios of angles 30° and 60°. For it can be shown that in this triangle the side opposite to angle of measure 60° is $\sqrt{3}$ times the length of the side adjacent to it. From the figure 6.39,

$\sin 30 = \frac{1}{2}$, $\cos 30 = \frac{\sqrt{3}}{2}$, $\tan 30 = \frac{1}{\sqrt{3}}$

$\sin 60 = \frac{\sqrt{3}}{2}$, $\cos 60 = \frac{1}{2}$, $\tan 60 = \sqrt{3}$

If in a right angled triangle one angle has measure 45° the other acute angle also has measure 45°, such a triangle is an isosceles triangle. Let the sides perpendicular to each other be of unit length. The hypotenuse then is of length $\sqrt{2}$ units, from fig. 6.40

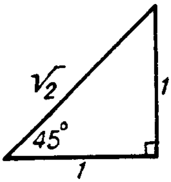


Fig. 6.40

$\sin 45 = \frac{1}{\sqrt{2}}$, $\cos 45 = \frac{1}{\sqrt{2}}$, $\tan 45 = 1$

A table of trigonometrical ratios of 30°, 60°, 45° is given below. We can use this table while solving problems.

Trigono-metrical-ratio	Measure of angle		
	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

§ 6.18 Solved Examples:

1. Find the value of $\sin^2 30 + \sin^2 45 + \sin^2 60$.

From the table $\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{1}{\sqrt{2}}$, $\sin 60 = \frac{\sqrt{3}}{2}$

$$\therefore \sin^2 30 + \sin^2 45 + \sin^2 60 = \frac{1}{4} + \frac{1}{2} + \frac{3}{4}$$

$$= \frac{1 + 2 + 3}{4}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

2. show that $\dfrac{1 - \tan^2 30}{1 + \tan^2 30} = \dfrac{1}{2}$

From table, $\tan^2 30 = (\frac{1}{\sqrt{3}})^2 = \frac{1}{3}$

$$\therefore \frac{1 - \tan^2 30}{1 + \tan^2 30} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{4} = \frac{1}{2}$$

3. Show that $\sin 30 . \cos 60 + \cos 30 . \sin 60 = 1$.

From table $\sin 30 = \frac{1}{2} = \cos 60$,

$\sin 60 = \frac{\sqrt{3}}{2} = \cos 30$

$$\therefore \sin 30 . \cos 60 + \cos 30 . \sin 60 = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

4. If $x \tan^2 60 + 3 \sin^2 60 - \frac{3}{4} \tan^2 30 = 6$, find x.

From table $\sin^2 60 = \frac{3}{4}$, $\tan^2 60 = 3$ and $\tan^2 30 = \frac{1}{3}$

$$\therefore x \tan^2 60 + 3 \sin^2 60 - \frac{3}{4} \tan^2 30 = x \times 3 + 3 \times \frac{3}{4} - \frac{3}{4} \times \frac{1}{3}$$

$$= 3x + \frac{9}{4} - \frac{1}{4}$$

$$= 3x + 2$$

$$\therefore 3x + 2 = 6$$

$$\therefore 3x = 4$$

$$\therefore x = \frac{4}{3}$$

5. Show that $1 + \tan^2 30 = \dfrac{1}{\cos^2 30}$

$\tan 30 = \frac{1}{\sqrt{3}}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$$\therefore 1 + \tan^2 30 = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\frac{1}{\cos^2 30} = \frac{1}{(\frac{\sqrt{3}}{2})^2} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\therefore 1 + \tan^2 30 = \frac{1}{\cos^2 30}$$

Also, $1 + \tan^2 \theta = 1 + \dfrac{\sin^2 \theta}{\cos^2 \theta}$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta}$$

Exercise 6.2

Find the values of.

1. $\sin^2 30 \times \cos^2 60$
2. $1 + \tan^2 30$
3. $1 - \sin^2 30$
4. $\tan^2 45 + \sin^2 60$
5. $\dfrac{\tan 60}{\sin 60 + \cos 60}$
6. $\dfrac{\tan 60}{\sin 30 - \cos 30}$
7. $2 \sin 30 . \cos 30$
8. $\cos^2 30 + \cos^2 45 + \cos^2 60$
9. $2 \tan^2 45 + 3 \tan^2 30 + 4 \tan^2 60$
10. $\sin 30 . \cos 60 + \cos 30 . \sin 60 + \tan 45$
11. $\sin 30 + \cos 60 - \frac{1}{2} \tan 45$
12. $\dfrac{1}{\sin^2 30} + \frac{5}{4} \sin^2 45 + \frac{3}{4} \cos^2 45$
13. $\dfrac{5 \sin^2 30 + 7 \cos^2 60 + 4}{8 \tan^2 45 + 3 + \tan^2 60}$
14. $3 \sin^2 \theta = 2 \frac{1}{4}$, from the table find θ.
15. $2x \tan^2 60 + 3x \sin^2 30 = \dfrac{27 \cos^2 45}{4 \sin^2 60}$, find x
16. $\tan^2 30 + \tan^2 45 + \tan^2 60$
17. $\cos 45 . \cos 60 + \sin 45 . \sin 60$
18. $\cos 60 . \cos 30 - \sin 60 . \sin 30$
19. $\cos 60 . \cos 30 + \sin 60 . \sin 30$
20. $\frac{4}{5} \tan^2 60 + 3 \sin^2 60 - 2 \left(\dfrac{1}{\sin^2 30} \right) - \frac{3}{4} \tan^2 30$
21. $\dfrac{\sin 30 + \cos 60}{2 \cos 60 - \sin 30} \times 2 \tan 45$
22. $3 \sin 30 - 4 \sin^3 30$

show that

23. $\sin^3 30 + \cos^3 30 = (\sin 30 + \cos 30)(1 - \sin 30 \cos 30)$
24. $\sin^3 30 - \cos^3 30 = (\sin 30 - \cos 30)(1 + \sin 30 \cos 30)$
25. $\sin^4 60 - \cos^4 60 = 1 - 2 \cos^2 60$
26. $\tan 60 = \dfrac{2 \tan 30}{1 - \tan^2 30}$
27. $\sin 60 = 2 \sin 30 . \cos 30$

28. $4 \tan^2 45 + \sin^3 30 - \dfrac{1}{\cos^2 60} = \dfrac{1}{8}$ **29.** $\cos 60 = \cos^2 30 - 1$

30.

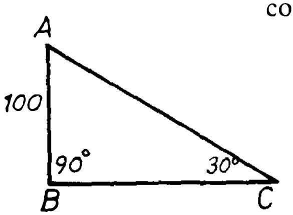


Fig. 6.41

In △ ABC $m \angle B = 90°$
$m \angle ACB = 30°$, $l(AB) = 100$
find $l(AC)$, $l(BC)$

31. As shown in the fig. 6.42
seg AB ⊥ seg BD. $m \angle ADB = 30°$
$m \angle ACB = 60°$. If $l(AB) = 50$ find
$l(BC)$, $l(DC)$.

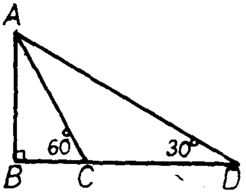


Fig. 6.42

32.

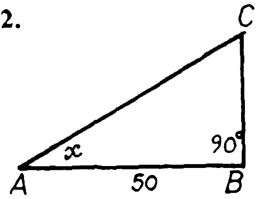


Fig. 6.43

In triangle ABC,
$m \angle ABC = 90°$, $m \angle CAB = x$,
$l(AB) = 50$, $\tan x = 1.32$
find $l(BC)$

33. In the fig. 6.44, $m \angle ABC = 90°$,
$m \angle CAB = 45°$, $m \angle CDB = 60°$
$l(AD) = 70$, find $l(BC)$
[**Hint :** use $\tan 45 = 1$, $\tan 60 \approx 1.73$ and the property of 30-60-90 triangle]

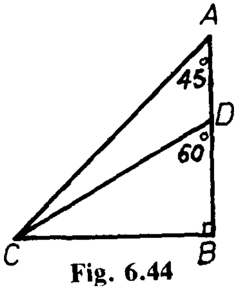


Fig. 6.44

§ 6.19 Dependence of the values of trigonometrical ratios on the size of the angles:

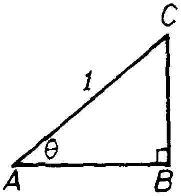


Fig. 6.45

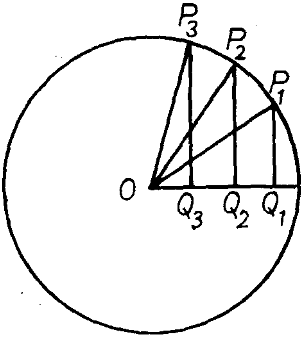


Fig. 6.46

In a right angled triangle let
$m \angle ABC = 90°$. $l(AC) = 1$ and
$m \angle BAC = \theta$, then $\sin \theta = l(BC)$
$\cos \theta = l(AB)$

In fig. 6.46, a circle with centre O and unit radius is shown.

$\therefore \sin \angle P_1OQ_1 = l(P_1Q_1);$

$\sin \angle P_2OQ_2 = l(P_2Q_2);$

$\sin \angle P_3OQ_3 = l(P_3Q_3)$

In the figure the larger acute angle has bigger side opposite to it and a smaller acute angle has smaller side opposite to it. So the value of sine ratio of an acute angle increases as it increases in measure.

similarly,

$$\cos \angle P_1OQ_1 = l(OQ_1)$$
$$\cos \angle P_2OQ_2 = l(OQ_2)$$
$$\cos \angle P_3OQ_3 = l(OQ_3)$$

In the figure the larger acute angle has smaller side adjacent to it and smaller acute has larger side adjacent to it. So the value of the cosine ratio of an acute angle decreases as the angle increases in measure.

Again,

$$\tan \angle P_1OQ_1 = \frac{l(P_1Q_1)}{l(OQ_1)}$$

$$\tan \angle P_2OQ_2 = \frac{l(P_2Q_2)}{l(OQ_2)}$$

$$\tan \angle P_3OQ_3 = \frac{l(P_3Q_3)}{l(OQ_3)}$$

In the figure as the acute angle increases the opposite side of the angle increases and the adjacent side decreases. Therefore as an acute angle increases the value of tangent ratio increases.

§ 6.20 Heights and distances:

Let us now see how trigonometrical ratios can be used to find heights and distances without being required to measure them physically.

From the point of view of mathematics a tree standing on ground is rougly speaking perpendicular to it. Distance between its base and the topmost end is its height. In mathematics it is also assumed that the top of tree is viewed from the level of the ground. The eye that views the top of the tree and its base are vertices of a right angled triangle. If in this right angled triangle one acute angle and the length of one of the sides are known then with the help of trigonometrical ratios all other distances become known. By now you must have realised this fact from the experience you have gained in solving problems of this type. Before we solve more examples let us get aquainted with some technical terms.

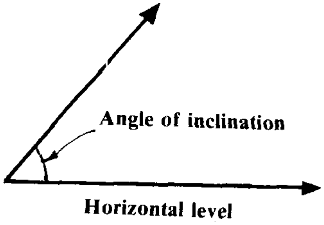


Fig. 6.47

In order to look at a given point a viewer has to raise his line of vision above the horizontal level so that he could properly view the given point. The angle which a ray joining eye to the point makes with the horizontal level is known as **angle of inclination or angle of elevation.**

When a viewer looks at an object on the ground while standing on a high tower or from the top of a tree he has to lower his line of vision below the horizontal level. The angle made by the ray that joins the eye to the object viewed makes with the horizontal level is known as **angle of declination or an angle of depression.**

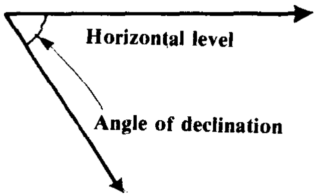


Fig. 6.48

One special point must however be noted in connection with the angle of inclination and angle of declination. Although we speak of the angle of inclination or declination of an object in reality the angle is made at the point where the eye is situated and not at the point at which the object is located. The statement that the angle of inclination of an aeroplane is 60° means that line of vision makes an angle of 60° with the horizontal level at the point where the eye is and not at the point where the aeroplane is. Let us now study some examples.

§ 6.21 Solved Examples:

1. The angle of inclination of the top of a tree when viewed from a point C at a distance of 20 m is 60°. What is the height of the tree ?

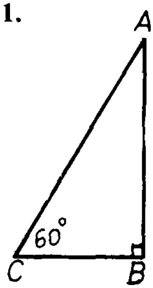


Fig. 6.49

In the diagram seg AB represents the tree. A is the top and B is the base. $d(B,C) = 20$ m; $m\angle ABC = 90°$, $m\angle BCA = 60°$. Let h be the height of the tree.

From the figure,

$$\tan 60 = \frac{h}{20}$$

$$\therefore \sqrt{3} = \frac{h}{20} \quad [\tan 60 = \sqrt{3}]$$

$$\therefore h = 20\sqrt{3}$$

$$= 20 \times 1.732 \ (\sqrt{3} \approx 1.732)$$

$$= 34.64 \text{ m.}$$

2. The angle of declination which a man is required to make while viewing at a stationary bus from the top of a building 30 m high is 30°. At what distance from the building is the bus standing ?

In the figure seg AB stands for the building. The bus is standing stationary at C. $l(AB) = 30$ m, $m\,\angle ABC = 90°$. A is the top and B is the foot of the building. The line of vision is along the ray AC. It makes an angle 30° with the horizontal level. So that $m\,\angle ACB = 30°$.

Let $x = d(B,C)$

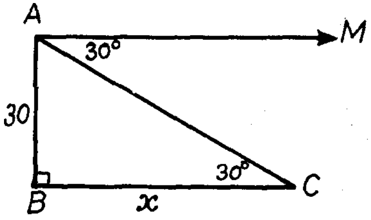


Fig. 6.50

From the figure,

$$\tan 30 = \frac{30}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{30}{x} \quad (\tan 30 = \frac{1}{\sqrt{3}})$$

$$x = 30\sqrt{3}$$

$$= 30 \times 1.732 \ (\sqrt{3} \approx 1.732)$$

$$= 51.96 \text{ m.}$$

3. At both sides of a road 10 m wide stand two buildings opposite to each other. One building is 30 m high. If from the top of this building the top of another is veiwed,the angle of inclination is 37°. What is the height of the other building ? Given that $\tan 37 \approx 0.754$.

In the figure seg AC and seg DB represent the two buildings. $l(AC) = 30$ m. A is the top of the first building and B is the top of the second. C and D are their feet respectively. If a horizontal line is drawn to intersect seg BD in E. $m\,\angle$ BAE = 37°. Let $l(BE) = h$, $l(ED) = 30$ m and $l(DC) = 10$ m.

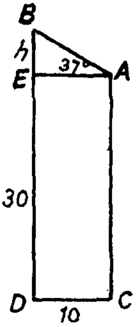


fig. 6.51

From the figure,

$$\tan 37 = \frac{h}{10}$$

$$\therefore 0.754 = \frac{h}{10}$$

$$\therefore h = 7.54 \text{ m.}$$

$\therefore$ The height of the other building $= 30 + 7.54$

$= 37.54$ m

4. While dashing to the destination point on the ground the pilot of the aeroplane declines his aeroplane by 20° and drives straight to the ground. The average speed of the plane in 200 km per hour. It takes 54 seconds to reach the ground. Before it started its dash how high was the aeroplane? $\sin 20 \approx 0.342$

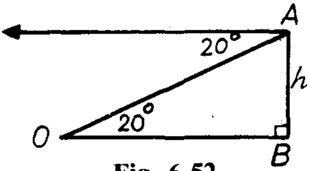


Fig. 6.52

In the diagram seg AB denotes the height h km of the aeroplane. O is the point of destination on the ground. In 60 minutes the aeroplane travels 200 km.

Therefore in $\frac{54}{60}$ minutes it will travel

$$\frac{200}{60} \times \frac{54}{60} \text{ km} = 3 \text{ km}$$

$$\therefore \; l(AO) = 3 \text{ km}$$

From the figure $\sin 20 = \frac{h}{3}$

$$\therefore \; 0.342 = \frac{h}{3}$$

$$\therefore \; h = 3 \times 0.342$$

$$= 1.026 \text{ km.}$$

5. In the figure 6.53, seg AB shows a tower with the top A and base B. If from points C and D in line with its base the angles of inclination of the top are respectively 45° and 30° and if distance between C and D is 30 m, find the height of the lower.

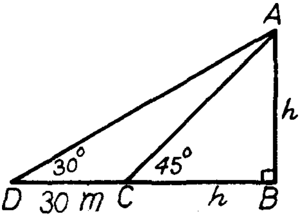


Fig. 6. 53

From the figure,

let $l(BC) = h$

$$\therefore \; \tan 30 = \frac{h}{h + 30}$$

$$\therefore \; \frac{1}{\sqrt{3}} = \frac{h}{h + 30}$$

$$\therefore \; h + 30 = \sqrt{3}h$$

$$\therefore \; 30 = h(\sqrt{3} - 1)$$

$$\therefore \; h = \frac{30}{\sqrt{3} - 1}$$

$$= \frac{30(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{30(\sqrt{3} + 1)}{2}$$

$= 15(\sqrt{3}+1)$

$= 15 \times 2.732 \quad (\sqrt{3} \approx 1.732)$

$= 40.980$ m

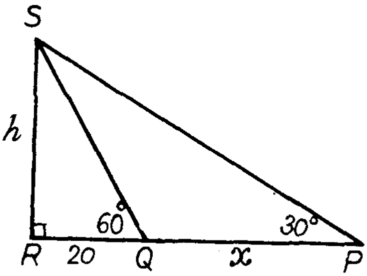


Fig. 6.54

6. In the adjoining figure 6.54, $m\angle P = 30°$ $m\angle R = 90°$, $m\angle Q = 60°$ $l(QR) = 20$.
Find $l(PQ)$ and $l(SR)$

Suppose that $l(SR) = h$; $l(PQ) = x$

Then from the two right angled triangles in the figure,

$$\tan 60 = \frac{h}{20} \qquad \tan 30 = \frac{h}{x+20}$$

$$\therefore \sqrt{3} = \frac{h}{20} \qquad \therefore \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\therefore h = 20\sqrt{3} \qquad \therefore \frac{1}{\sqrt{3}} = \frac{20\sqrt{3}}{x+20}$$

$$\therefore x + 20 = 20\sqrt{3} \times \sqrt{3} = 60$$

$$\therefore x = 40$$

$\therefore l(PQ) = x = 40$ m.

$l(SR) = h = 20\sqrt{3} = 20 \times 1.732$

$= 34.64$ m.

7. In order to find the width of a river a man on one of its banks views the top of a tree just across the river on the other bank. His line of vision makes an angle of inclination of 61°. The man again views the top by receding 50 metres away from the bank in line with the base of the tree and his first position. This time angle of inclination is 35°. Find the width of the river and the height of the tree.

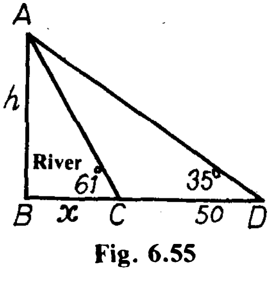


Fig. 6.55

$\tan 35 = 0.7$; $\tan 61 = 1.8$

Let seg BC denote the width of the river. seg AB the height of the tree, B-C-D and $d(B,C) = x$, $d(C,D) = 50$ m.

From the figure,

$$\tan 61 = \frac{h}{x}; \qquad \tan 35 = \frac{h}{x+50}$$

$$\therefore \frac{h}{x} = 1.8 \text{ and } \frac{h}{x+50} = 0.7$$

$$\therefore\ h = 1.8x$$

$$\therefore\ \frac{h}{x+50} = \frac{1.8x}{x+50} = 0.7$$

$$\therefore\ 1.8x = 0.7x + 0.7 \times 50$$

$$\therefore\ 1.1x = 35.0$$

$$\therefore\ 11x = 350$$

$$\therefore\ x = \frac{350}{11} = 31.82 \text{ m}$$

$$\begin{aligned} h &= 1.8x \\ &= 1.8 \times 31.82 \\ &= 57.276 \text{ m} \end{aligned}$$

Exercise 6.3

1. From a certain place on the bank of a river, the angle of elevation of the top of a temple which is standing on right opposite bank of the river in 30°. If the height of the temple is 20 m. find the breadth of the river.
2. A ladder on a fire truck can be extended to maximum length of 20 m when elevated to its maximum angle 70°. The base of the ladder is mounted on the truck 2 metres above the ground. How high above the ground will the ladder reach ? [$\sin 70 \approx 0.940$]
3. A peacock is sitting on the top of a tree. It observes a serpent on the ground making angle of depression of 25°. The peacock with the speed of 300 m per minute catches the serpent in 12 sec. what is the height of the tree ? [$\sin 25 \approx 0.423$]
4. A forest ranger watches for fire from a lockout tower built on a high hill. The height of the tower is 80 m above most of the surrounding land and the tower itself is 15 m tall. If the ranger sights a fire at an angle of depression of 10° with the horizontal how far is the fire from the tower ? [$\tan 10 \approx 0.176$]

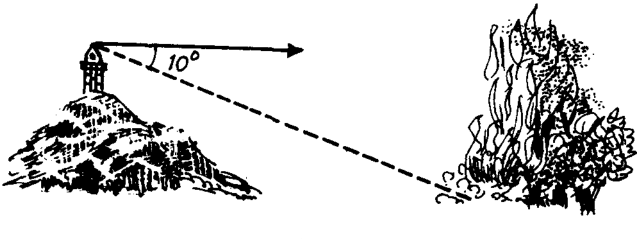


Fig 6.56

5. Due to heavy wind a tree is broken. The end of the broken part touches the ground at a point which is 8 metres from its trunk. If the broken part of the tree makes an angle 54° with the ground, find the height of the unbroken tree. [$\cos 54 \approx 0.588$, $\tan 54 \approx 1.376$]

6. The top of the tower 60 metre high makes the angles of depression 30° and 60° with the top and the bottom of the coconut tree respectively. Find the height of the tree and the distance between the bottoms of the tower and the tree. [$\sqrt{3} \approx 1.732$]

7. P and Q are two points on the opposite banks of a river such that line PQ is perpendicular to the banks of the river. When a person from the top of a tower 50m high, looks at the points P and Q his line of vision makes angles of depression 45° and 30° respectively. If the base of the tower and the points P and Q are in one line, find the breadth of the river.

8. A flagpost is erected on the top of a building. When a person standing at a distance of 15 m. from the building looks at the top and the bottom of the flagpost, his line of vision makes angles of elevation of 60° and 45° respectively. Find the the height of the flagpost.

9. A balloon sent in the atmosphere to collect the information about weather is steady at a certain height. As shown in the figure 6.57 from a point on the ground the angle of elevation is 25°. By moving 1 km. away from this point the angle of elevation changes to 15°. Find the height of the balloon from the ground.

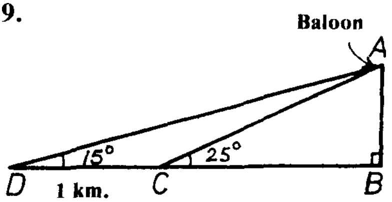


Fig. 6.57

[$\tan 15 \approx 0.268$; $\tan 25 \approx 0.466$]

10. When seen from the top and the bottom of the tower towards the top of a building, the angle of depression and the angle of elevation are 27° and 63° respectively. If the height of the building is 20 m, find the height of the tower. [$\tan 27 \approx 0.50$, $\tan 63 \approx 1.96$]

Miscellaneous Excercise - 6

1. With reference to the figure 6.58 complete the following statements.

   (1) sin B = ________

   (2) tan ∠ MCB = ___

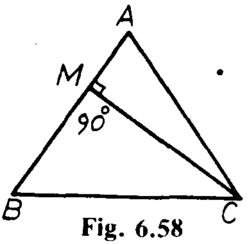


Fig. 6.58

2.

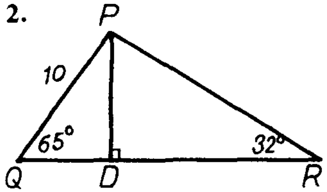


Fig. 6.59

In the figure 6.59 $l(PQ) = 10$ and $m\angle Q = 65°$, seg PD $\perp$ seg QR find $l(PD)$.

[sin 65 $\approx$ 0.906]

If $m\angle R = 32°$, then show that $l(PR) = 17.1$; also find $l(QD)$, $l(DR)$ and $l(QR)$. [sin 32 $\approx$ 0.530]

3. In Δ ABC, $l(BC) = 16$, $l(AB) = 10$ and $m\angle B = 57°$. Find the length of the perpendicular segment from vertex A on side BC. Also find the length of the perpendicular segment from vertex C to side BA.
[sin 57 $\approx$ 0.839]

4. In Δ DEF, $l(DE) = 8$, $l(EF) = 12$ and $m\angle E = 48°$; Find the area of Δ DEF. (sin 48 $\approx$ 0.743)

5. Fill in the gaps in the following statements with reference to the adjoining fig. 6.60

(A) (1) sin $\angle$ ABD = cos $\angle$ B ......D
(2) sin $\angle$ ABD = sin $\angle$ ....D
(3) cos $\angle$ DBC = cos $\angle$ B.......

(B) Fill in the gaps in the following statements using one of the symbols $=, <, >$:
If $m\angle$ ABD $>$ $m\angle$ BCD,
then sin $\angle$ DBC .....sin $\angle$ BCD

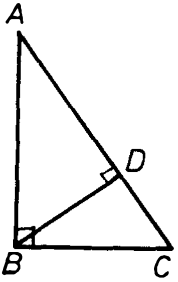


Fig. 6.60

6.

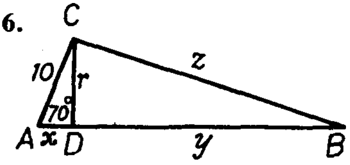


Fig. 6.61

In the figure, in Δ ABC, $l(AC) = 10$, $m\angle ACB = 90°$ seg CD $\perp$ seg AB and $m\angle CAB = 70°$.
Find x = ______; y = ______
r = ______; z^2 = ______
(sin70° $\approx$ .940 , tan70° $\approx$ 2.75)

7. In Δ PQR, $l(PQ) = 12$, $l(QR) = 15$ and $m\angle Q = 60$. Show that $l(PR) = 13.75$

8. If $\sin x = \frac{8}{17}$ then $\cos x$ = ...; $\tan x$ = ...

9. show that $\left[\frac{1}{\sin\theta} + \frac{1}{\tan\theta}\right]^2 = \frac{1+\cos\theta}{1-\cos\theta}$

10. show that $\cos^2\theta\,(1 + \tan^2) = 1$

11. show that $4\,[\sin^4 30 + \cos^4 60] - 3\,[\cos^2 45 - 1] = 4$

12. show that $\dfrac{\tan x}{\dfrac{\sin^3 x}{\cos x} + \sin x \cdot \cos x} = 1$

13. Show that, $\sin \theta \cdot \cos(90 - \theta) + \cos \theta \sin(90 - \theta) = 1$

14. Show that, $\cos \theta \cos(90 - \theta) - \sin \theta \sin(90 - \theta) = 0$

15. Find the value of $3 \sin^2 30 + 2 \tan^2 60 - 5 \cos^2 45$

16. If $m \angle A = 60°$, $m \angle B = 30°$ then verify that

(1) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(2) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

17. If $\sin A = \frac{7}{25}$ then find the value of $\sin A(1 + \sin A) + \cos A(1 + \cos A)$

18. Find the value of $\cos 38 + \tan 45 - \sin 52$.

19. Find the value of $\dfrac{5 \sin^2 30 + 7 \cos^2 60 + 4}{8 \tan 45 + 3 + \tan^2 60}$

20. Show that $\sin^2 37 + \sin^2 53 = 1$

21. What is the angle of elevation of the sun when a post of height 9 metres casts a shadow of length $3\sqrt{3}$ m. ?

22. The angle of elevation of a point A with the top of the hill is 60°. B is the point at a distance of 20 m. Vertically above A. If the angle of elevation of the point B with the top of the hill is 30°, find the height of the hill and the distance of the point A from the bottom of the hill.

23. The angle of elevation of a point P with the top of a mountain is 45°. B is a point at a distance of 2 km towards the mountain on a straignt road whose inclination with the horizontal is 30°. If the angle of the inclination of the point B with the top of the mountain is 60°, find the height of the mountain.

□ □ □ □

Chapter 7

MENSURATION

§ 7.1 In your nineth standard you made a study about areas of plane figures. This year your interest of study will mainly be about volumes of a right circular cylinder and a right circular cone. For this let us revise some fundamental notions.

Area of a closed plane figure is the amount of space (in the plane) enclosed by that figure. The space enclosed by a square of unit side is taken as a unit for mesuring areas. To find the area enclosed by a plane figure, then is to find, the number of such unit squares that will exactly cover the space enclosed by the figure.

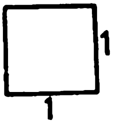


Fig. 7.1

You are acquainted with various fomulae for determining the areas of various plane figures. You know that when the radius of a circle is R units, its area is

$$\pi R^2 \text{ (sq units).}$$

In this chapter, we, now, consider three dimentional figures. Such figures occupy some part of the three dimesional space. The space occupied by three dimensional figures is measured in terms of cubic units and is called its volume.

You know what a cube is.

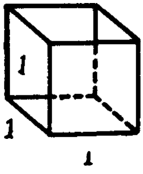


Fig. 7.2

The volume of a cube of side one unit is taken as a unit for measuring volumes. It is the amount of space occupied by a cube. To determine the volume of a three dimensional figure is to find out the number of unit cubes which will exactly fit the portion of space occupied by that figure.

But the problem of determining such a number is not as simple as it appears on the face value. To be able to answer this question in respect to all three dimensional figures, mathematics as a science, had to attain a certain stage of development. The branch of mathematics known as integral calculus has its origin in man's attempt to tackle this problem successfully.

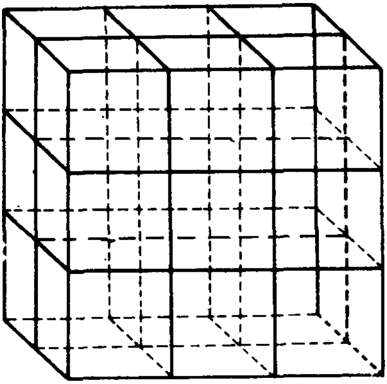


Fig. 7.3

In this chapter we will study only a specific number of three dimensional figures.

As in a cube some three dimensional figures are bounded by planes. The edges of such figures are formed by intersection of two planes. The point of intersection of two edges is the vertex of these figures.

Let us assume that there is a polygonal region in the plane of the paper. Assume that this plane region is moved parallel to itself in a direction perpendicular to the plane. The three dimensional figure so formed is known as a right cylinder. All but two faces of such a right cylinder are rectangles This right cylinder stands between two parallel planes.

The length of any edge of this right cylinder is known as the height of the cylinder. The polygonal region in the two parallel planes are respectively known as lower and upper base of the right cylinder. These right cylinders are named after the names of the polygonal figures. In figures shown below are right triangular cylinder or prism, right pentagonal cylinder, right hexagonal cylinder and a rectangular parallelepiped.

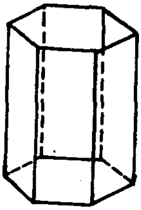
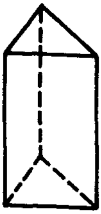
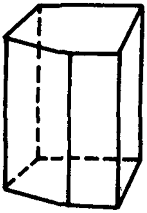
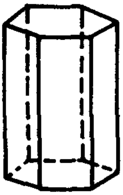
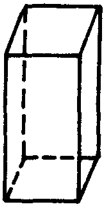


Fig. 7.4

			
7·5	7·6	7·7	7·8
Prism	**Right pentagonal Cylinder**	**Right hexagonal Cylinder**	**Rectangular-Parallele-piped**

The plane faces of right cylinders fall into two classes, base faces and vertical faces. Since vertical faces are rectangular, their areas can be found out easily. The areas of base faces together with the area of vertical faces is the total surface area of the right cylinder. The right cylinder has the same number of vertical faces as the number of sides of the polygonal bases. By adding 2 to this number we know the number of plane faces bounding the given right cylinder. Triangular right cylinder or a prism has five faces. Pentagonal right cylinder has seven faces and hexagonal right cylinder has eight faces.

If the total number of faces is denoted by F, the number of edges is denoted by E and the number of vertices is denoted by V, then by Euler's formula

$$F + V = E + 2$$

This formula you have studied in your VIII standard.

With the help of this formula fill up the following table.

Sr. No.	Name of the figure	Number of faces	Number of vertices	Number of edges
1	Triangular right cylinder	—	—	—
2	Hexagonal right cylinder	—	—	—
3	Decagonal right cylinder	—	—	—
4	Octagonal right cylinder	—	—	—
5	——	6	8	—
6	——	7	—	15
7	——	10	16	—
8	——	—	18	27

§ 7.2 Volume of a right cylinder:

A right rectangular cylinder or a rectangular parallelepiped has two rectangular bases. Let the length of the rectangle be L and breadth be B and the height of the right cylinder or rectangular parallelepiped be H. Therefore the area of each base is $A = L \times B$

The volume of the rectangular cylinder will be $= L \times B \times H$

$\therefore$ Volume of a rectangular cylinder $= L \times B \times H$

$= A \times H$

The formula: Area of the base × height, for the volume of a right rectangular cylinder can be applied to any right cylinder. Let us denote the height of a right cylinder by H and area of its base by A. Then,

Volume of a right cylinder $= A \times H$

Exercise 7.1

1. Volume of a right cylinder is 500 cu cm and its height is 10 cm. Find the area of its base.
2. A hall is of 20 m in length and 15 m in breadth. If the volume of the hall is 3000 cu m, find its height.
3. The height of a rectangular cylinder is 10 cm and its area of surface other than base faces (area of the vertical faces) is 52 sq cm find the perimeter of its base.
4. The perpendicular sides of the base of a right triangular prism are 6 cm and 8 cm. If the height of the triangular prism is 25 cm, find its volume.
5. The volume of a right cylinder is 160 cu cm and its height is 4 cm. The base of the right cylinder is a trapezium, one of whose parallel sides is 6 cm and perpendicular distance between the parallel sides is 5 cm. Find the length of the other parallel side of the trapezium.
6. The length of a side of a regular hexagonal pillar is 2 m and the height of the pillar is 15 m. Find its volume.
7. The perimeter of a regular hexagonal right cylinder is 60 cm and its height is 20 cm. Find its volume and the area of the vertical faces.

§ 7.3 Right circular cylinder:

In fig. 7.9, the bases of a right cylinder are circular regions. Let us assume, there is a circular region in the plane of the paper. Assume that this plane is moved parallel to itself in a direction perpendicular to the plane of the paper. The figure so formed is known as a **right circular cylinder.** This three dimensional figure stands between these two planes. The distance between the two planes is known as the height of the right circular cylinder. If from a point of a circle of the upper base a perpendicular segment is drawn to the lower base, it lies entirely on the curved surface of the right circular cylinder.

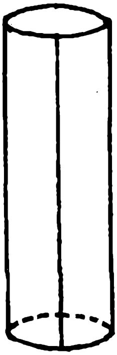


Fig. 7.9

The length of this segment which is also the distance between the two parallel planes between which, the cylinder stands, is the height of the right circular cylinder.

The formula, Area of base × Height is also applicable to the right circular cylinder. Let us denote the height by H and area of its base by A

Then the volume of the right circular cylinder $= A \times H$

If radius of the base of a right circular cylinder be R, then the area of its base is πR^2

∴ Volume of the height circular cylinder = $\pi R^2 H$

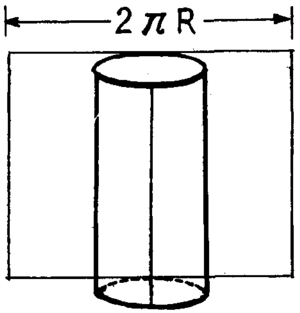


Fig. 7.10

A right circular cylinder has two plane bases and a vertical face which is not plane but curved. We shall call this as the curved surface of the right circular cylinder. If the radius of the circular base is R then the length of its circumference is $2 \pi R$. If you cut the surface vertically along its height and spread over the surface in a plane, you will get a rectangular region. The length of this rectangle is $2 \pi R$ and its breath is H.

If this rectangular region again covers the curved surface of the cylinder, it will do so completely.

∴ Curved surface area of a right circular cylinder = $2 \pi R \times H$

Area of two bases of the right circular cylinder = $2 \times \pi R^2$

∴ The total surface area of the right circular cylinder

$$= 2 \pi R^2 + 2 \pi RH$$

$$= 2 \pi R (R + H)$$

Let us denote the volume of the circular cylinder by V, the area of curved surface by S_c and its total surface area by S_t

$$\therefore V = \pi R^2 H \quad ...(1)$$

$$S_c = 2 \pi RH \quad ...(2)$$

$$S_t = 2 \pi R (R + H) ...(3)$$

Each formula contains three variables.

If values of any two variables are known the corresponding value of the third variable can be determined. For example in the formula (3), if S_c and R are known H can be found out. Similarly in the first formula if V and H are given R can be determined.

§ 7.4 Solved Examples

1. The radius of the base of a right circular cylinder is 14 cm and its height is 20 cm. Find the area of its curved surface and also the total surface area.

Solution

S_c = Area of the curved surface of the right circular cylinder

$= 2\pi RH$

$= 2 \times \frac{22}{7} \times 14 \times 20$ sq cm

$= 1760$ sq cm

S_t = Total surface area of the right circular cylinder

$= 2\pi R(R + H)$

$= 2 \times \frac{22}{7} \times 14(14 + 20)$

$= 2992$ sq cm

2. 70 right circular cylindrical boxes are to be prepared from tin sheets. The radius of the base of each box is 15 cm and the height is to be 30 cm. How many square metres of tin sheets are required ?

Solution: To prepare a right circular cylindrical box closed at both ends, the tin sheets required are equal in area to the total surface area.

$S_t = 2\pi R(R + H)$

$= 2 \times \frac{22}{7} \times 15(15 + 30)$

$= \frac{22 \times 30 \times 45}{7}$

For a box $\frac{22 \times 30 \times 45}{7}$ sq cm of tin sheets are required.

∴ To prepare 70 boxes

$\frac{70 \times 22 \times 30 \times 45}{7}$ sq cm of tin sheets are required.

i.e. $\frac{70 \times 22 \times 30 \times 45}{7 \times 10000}$ sq m of tin sheets are required.

i.e. 29.70 sq m of tin sheets.

3. The diameter of a road roller is 1 metre and its length is 1.4 m. In 100 revolutions how much land will be pressed ? And how many revolutions are required to press 11000 sq m of land once ?

Solution:

Here, we shall have to find the surface area of the road roller. Again, here $R = 0.5$ m and $H = 1.4$ m.

∴ $S_c = 2\pi RH$

$= 2 \times \frac{22}{7} \times 0.5 \times 1.4$

$= 2.2$ sq m

∴ In 100 revolutions of road roller, 100 × 2.2 sq m i.e. 220 sq m land will be pressed.

Again to press 2.2 sq m land the road roller is required to make one revolution.

∴ To press 11000 sq m of land $\frac{11000}{2.2}$

i.e. 5000 revolutions are required.

4. The outer and inner radii of a right circular cylinder type tank are respectively 1.2 m and 1 m; The height of the tank is 2 m. It is closed from one side. Find the expenditure required to paint the inner and outer surfaces of the tank at the rate of Rs. 7/- per sq m (The rim not to be considered.)

The inner surface area of the tank

= Inner curved surface area of the tank + Area of the base

$= 2\pi RH + \pi R^2$

$= \pi R(2H + R)$

$= \frac{22}{7} \times 1(4 + 1)$ sq m

[Here R = 1 m, H = 2 m]

$= \frac{110}{7}$ sq m ...(1)

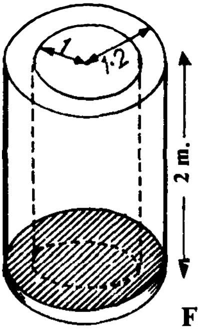


Fig. 7.11

The outer surface area of the tank

= The outer curved surface area + Area of the base.

$= 2\pi RH + \pi R^2$ [Here R = 1.2 m, H = 2 m]

$= \pi R(2H + R)$

$= \frac{22}{7} \times 1.2(4 + 1.2)$

$= \frac{22}{7} \times 1.2 \times 5.2$

$= \frac{137.28}{7}$ sq m ...(2)

The total surface area to be painted

$= \frac{137.28}{7} + \frac{110}{7}$ sq m (from (1) and (2))

$= \frac{247.28}{7}$

∴ The total expenditure to paint the surface area of the tank

$= 7 \times \frac{247.28}{7} = 247.28$ Rs.

Exercise 7.2

1. If radius R, the height H of a right circular cylinder are given, find the curved surface area (S_c) and total surface area (S_t) of the cylinder. [$\pi \approx \frac{22}{7}$]
   (*a*) R = 14 cm. and H = 5 cm.
   (*b*) R = 21 cm. and H = 15 cm.
   (*c*) R = 56 cm. and H = 2 m.
   (*d*) R = 8.4 dm. and H = 2.5 m.
   (*e*) R = 1.4 m. and H = 2.8 m.
2. The circumference of the base of a right circular cylinder is 26.4 cm and its height is 15 cm. Find its total surface area.
3. The height of a right circular cylinder is equal to the circumference of the base whose radius is 2.1 cm. Find its curved surface area and its total surface area.
4. The length and the breadth of a rectangular tin sheet are 3.85 m and 2 m respectively. From this tin sheet circular pipes are prepared of length 50 cm and radius of their cross-section being 35 cm. How many pieces of pipe can be prepared ?
5. A building has 8 right circular cylindrical pillars whose cross-sectional radius is 1 m and whose height is 4.2 m. Find the expenditure to paint these pillars at the rate of Rs 2 per square meter.
6. The breadth of a rectangular tin sheet is 1.65 m and its length is 5 m. A pipe is prepared from this tin sheet. The radius of whose circular cross section is 35 mm. Find its length in metres.
7. The outer and inner diameters of a circular pipe of length 10.5 m are 1.2 m and 1 m respectively. Find the expenditure to paint both the inner and outer surfaces of the pipe at the rate of Rs 1.00 per square meter.
8. The area of the curved surface of a right circular cylinder is 770 sq cm If the radius of its base is 1 cm, find its height.
9. A right circular cylindrical vessel of 0.2 m. radius and 0.70 m. in height is to be nickel plated. What is the total cost of plating @ Rs. 7.00 per sq m ?
10. A thermal chimney of circular cross-section has outer and inner radii 3 m and 2 m respectively. Its height is 56 m. Find the expenditure for cement finishing @ Rs. 20.00 per square meter.
11. A road roller is of diameter 1.75 m and of length 1 m. How much ground can be pressed by it in its 200 revolutions ?

12. The length and the breadth of a rectangular play ground are 110 m and 60 m respectivley. The diameter of a road roller is 1 m and its length is 1.75 m. If the whole playground is to be pressed once how many revolutions will this road roller make ?

13. The length and the breadth of a rectangualar tin sheet are 4.40 m and 1.16 m respectively. In preparing closed right circular cylindrical tin sheet box, 88 sq cm tin sheet is wasted. If the radius of the base is to be 7 cm and its height is to be 20 cm, how many boxes can be made from this tin sheet ?

14. The total surface area of a right circular cylinder is 158.84 sq m If its diameter is 3.5 m, find its height.

15. Find the total surface area of a right circular cylindrical pillar whose height is 3 m and whose circumference is 132 cm.

§ 7.5 Solved Problems:

1. The circumference of a right circular cylinder is 4.4 cm and its height is 10 cm. Find its volume.

Solution:

$$\text{Circumference} = 2\pi R$$

$$\therefore 2 \times \frac{22}{7} \times R = 4.4$$

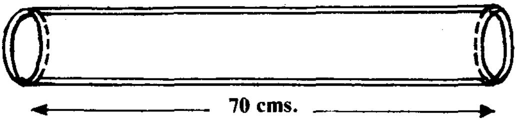
$$\therefore R = 0.7 \text{ cm.}$$

Here $R = 0.7$ cm and $H = 10$ cm. Volume V of right circular cylinder is given by $V = \pi R^2 H$

$$= \frac{22}{7} \times 0.7 \times 0.7 \times 10$$

$$= 15.4 \text{ cu cm.}$$

2. The outer and inner diameters of a circular cross-section of a pipe are 8 cm and 6 cm respectively. Its length is 70 cm. By melting the pipe a right circular cylindrical rod of 1 cm thickness is prepared. Find its length.



Solution:

To find the volume of the circular pipe we must find the volume of the right circular cylinder of outer radius and deduct the volume of the right circular cylinder of inner radius.

(1) $V_1 = \pi R^2 H$

$= \frac{22}{7} \times 4 \times 4 \times 70$ [Here outer radius $= \frac{8}{2} = 4$]

(2) $V_2 = \frac{22}{7} \times 3 \times 3 \times 70$ [Here inner radius $= \frac{6}{2} = 3$]

$\therefore$ Volume of the pipe $= V_1 - V_2$

$= 22 \times 160 - 22 \times 90$

$= 1540$ cu cm

We have to prepare a right circular cylindrical rod whose radius is 0.5 cm and whose volume is 1540 cu cm

Volume of the rod = Volume of the pipe

$\therefore \pi R^2 H = 1540$

$\therefore \frac{22}{7} \times 0.5 \times 0.5 \times H = 1540$

$\therefore H = 1540 \times \frac{7}{22} \times \frac{2}{1} \times \frac{2}{1}$

$= 1960$ cm.

$\therefore H = 19.60$ m.

3. From the material of a right circular cylindrical,copper rod of 7 cm cross-sectional diameter and 2 m in length, a 2 mm thick wire is made. What is its length ?

Solution:

Let the length of the wire be h cm.

Volume of the wire = Volume of the copper rod.

$\therefore \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 200$

$\therefore h = 245000$ cm

$h = 2450$ m

4. From a tap whose inner radius is 3.5 cm water flows at the rate of 10 m per second. If this tap is opened in a right circular cylindric tank whose diameter is 2 m and whose height is 4.9 m, find the time required to fill this tank.

Solution:

Volume of the tank $= \pi R^2 H$

$= \frac{22}{7} \times 1 \times 1 \times \frac{49}{10}$

$= 15.4$ cu m

The volume of water flowing out in one second from the tap

$= \frac{22}{7} \times \frac{35}{1000} \times \frac{35}{1000} \times 10$ [Here R = 3.5 cm $= \frac{35}{1000}$ m]

$= \frac{385}{10000}$ cu m

$\therefore$ Time required to fill the tank $= \dfrac{\text{volume of tank}}{\text{volume of water flowing in one second.}}$

$$= \frac{\frac{154}{10}}{\frac{385}{10000}}$$

$= 400$ sec.

$= 6$ minutes, 40 seconds.

5. Volume of a right circular cylinder is 1540 cu cm and area of its curved surface is 440 sq cm. Find its radius and height.

$$V = \pi R^2 H = 1540 \text{ and } S_c = 2 \pi R H = 440$$

$$\therefore \frac{V}{S_c} = \frac{\pi R^2 H}{2 \pi R H} = \frac{1540}{440}$$

$$\therefore \frac{R}{2} = \frac{7}{2}$$

$$\therefore R = 7 \text{ cm.}$$

$$S_c = 2 \pi R H = 440$$

$$\therefore 2 \times \frac{22}{7} \times 7 \times H = 440$$

$$\therefore H = 440 \times \frac{1}{2 \times 22}$$

$$H = 10 \text{ cm.}$$

6. Volume of a right circular cylinder is half of its total surface area. If the radius of its base is 3.5 cm, find its height, volume and curved surface area.

Solution:

$$V = \pi R^2 H \text{ and } S_t = 2 \pi R(R + H)$$

$$\therefore \frac{V}{S_t} = \frac{1}{2} = \frac{\pi R^2 H}{2 \pi R (R + H)}$$

$$\therefore \frac{\frac{22}{7} \times (3.5)^2 \times H}{2 \times \frac{22}{7} \times (3.5 + H)} = \frac{1}{2}$$

$$\therefore \frac{3.5 H}{2(3.5 + H)} = \frac{1}{2}$$

$$\therefore 3.5 H = 3.5 + H$$

$$\therefore 2.5 H = 3.5$$

$$\therefore H = \frac{3.5}{2.5} = \frac{7}{5} \text{ cm}$$

$$\begin{aligned} V &= \pi R^2 H \\ &= \frac{22}{7} \times (3.5)^2 \times \frac{7}{5} \\ &= 53.90 \text{ cu cm} \\ S_c &= 2 \pi R H \\ &= 2 \times \frac{22}{7} \times 3.5 \times \frac{7}{5} \\ &= 30.8 \text{ sq. cm.} \end{aligned}$$

Exercise 7.3

1. The right circular cylinder has radius of its base as R, its height is H and its volume is V. Fill in the gaps in the table using the given information.

Sr. No.	R	H	V
(*a*)	3.5 cm	6 cm	—
(*b*)	2 cm	1.4 cm	—
(*c*)	1 m	7 dm	—
(*d*)	35 cm	—	173250 cu cm
(*e*)	—	5 cm	3080 cu cm

2. The weight of a right circular cylindrical metal piece is 297 gm. If the radius of its base is 1 cm and its length is 7 cm; find the density of the metal.
3. There are two vessels. One is of square base of side 8 cm and of height 13 cm. Another vessel is a right circular cylinder whose radius of base is 3.5 cm and whose height is 13 cm. Which vessel will contain more water ?
4. How many litres of petrol will be contained in a right circular cylindrical tank area of whose base is 9.625 sq.m. and whose height is 4 m ?
5. Area of the curved surface of a right circular cylinder is 3168 sq cm and its height is 21 cm Find its volume.
6. The total surface area of a right circular cylinder is 341 sq cm and radius of its base is 3.5 cm. Find its volume.

7. The circumference of a right circular cylinder is 88 cm and its height is 15 cm Find its volume.
8. The expenditure incurred to paint a right circular cylindrical wooden pillar at the rate of 1 paisa per sq cm is Rs 220.00. The radius of its base is 35 cm. The cost of the wood is Rs 2560/- per cubic meter. What is the cost of the pillar ?
9. A water tap is 10 m long. Its outer and inner diameters are 2 cm and 1.6 cm respectively. Find its volume.
10. Water flows at the rate of 7 m per second through a water tap whose cross sectional radius is 2 cm. This tap is opened in a right circular cylindrical tank whose height is 3.5 m and whose circumference is 6.16 m. Find the time required to fill up the tank.
11. The outer and inner circumferences of a tap are 6.6 cm and 4.4 cm respectively. If the volume of the tap is 1925 cu cm. Find its length.
12. A piece of a metal in the form of a right rectangular cylinder of dimensions 55 cm × 40 cm × 28 cm is converted into a right circular cylinder whose base diameter is 5.6 cm. Find its height.
13. To store grain there is an earthen right circular cylindrical tank. Its radius is 1.5 m and its height is 2.8 m. If 0.45 cu m grain can be stored in a gunny bag, find how many bags of grain can be stored in the tank.
14. A right cylindrical vessel is filled with milk. The height of the vessel is 50 cm and the radius of the circular cross-section is 21 cm. By means of a right circular cylindrical pot whose diameter is 7 cm and whose height is 10 cm, the milk from the vessel is to be distributed to students. How many students can be supplied with milk, if each student is to get a pot of milk ?
15. A right circular cylindrical piece of metal, whose base diameter is 24 cm and whose height is 31.4 cm is melted and pieces of right circular cylinder whose diameter is 4 cm and whose height is 7.85 cm are made. How many such pieces can be made ?
16. Through a pipe of diameter 110 mm water flows at the rate of 2 m per second. To wet one hector land 3593.7 cu m of water is required. What time is required to wet one hector of land ?

17. Volume of a right circular cylinder is half of its total surface area. If the radius of its base is 7 cm find its volume and the curved surface area.
18. Volume of a right circular cylinder is 550 cu cm and area of whose curved surface is 220 sq cm. What are its radius and height ?

§ **7.6** In a right angle triangle ABC, the sides AB and BC are perpendicular to each other and side AC is the hypotenuse. Suppose there is a play-card of this figure. Keeping side AB fixed that is assuming it as an axis if the figure is revolved completely once about side AB then the three dimentional figure so formed is a **right circular cone.** You are already acquainted with this three dimentional figure.

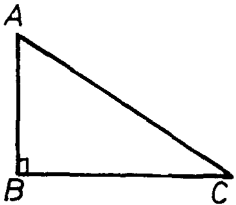


Fig. 7.13

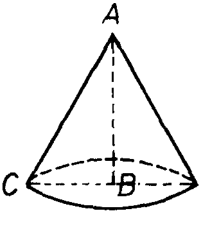


Fig. 7.14

By revolving the side BC about side AB we get a circular plane surface. Therefore the base of this three dimentional figure is a circular region. The fixed side AB is called the height of this right circular cone. The length of hypotenuse is known as the slant heigh of this right circular cone.

Let the slant height be denoted by L, the height by H and the radius of base by R. Since the slant height is the length of the hypotenuse, we have

$$L^2 = R^2 + H^2$$

Surface area of a right circular cone:

The right circular cone has two surfaces. One is a circular base and other is a curved surface. We can find the area of the curved surface of the right circular cone but here we shall use its formula without giving its proof. Let us denote the curved area of the right circular cone by S_c and its total surface area by S_t.

The total curved surface area of the right circular cone

$$= \frac{1}{2} \times \text{length of the cicumference} \times \text{its slant height}$$

$$= \frac{1}{2} \times 2 \pi R \times L$$

$$= \pi R L \quad ...(1)$$

The area of the total surface of this right circular cone

$$= \pi R^2 + \pi R L$$

$$= \pi R(R + L) \quad ...(2)$$

Volume of a right circular cone:

Volume of a right circular cone is related to the volume of the right circular cylinder having equal base and equal height. The volume of the cone is one third of the cylinder. If R is the radius of the circular base of the cone, H is its height and V is its volume, we have.

$$V = \frac{1}{3} \pi R^2 H \quad ...(3)$$

§ 7.7 Solved examples:

1. The base of a right circular cone has its radius 3 cm and the height of the cone is 4 cm. Find its curved surface area, its total surface area, and its volume. $[\pi \approx 3.14]$

Solution:

$R = 3$ cm $\quad H = 4$ cm (given)

$L^2 = R^2 + H^2$

$= 3^2 + 4^2$

$L = \sqrt{25} = 5$ cm

$S_c = \pi R L$

$= 3.14 \times 3 \times 5$

$\therefore S_c = 47.10$ sq cm ...(i)

$S_t = \pi R (R + L)$

$= 3.14 \times 3 (3 + 5)$

$= 3.14 \times 24$

$S_t = 75.36$ sq cm ...(ii)

$V = \frac{1}{3} \pi R^2 H$

$= \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$

$\therefore V = 37.68$ cu cm ...(iii)

2. The circumference of the base of a right circular cone is 22 cm and its height is equal to the diameter of its base. Find its volume and its total surface area.

Solution:

Circumference of the base $= 2\pi R$

$\therefore 22 = 2 \pi R$

i.e. $2 \times \frac{22}{7} \times R = 22$

$\therefore \quad R = \frac{7}{2}$ cm.

$\therefore$ Diameter of the base $= 2R = 2 \times \frac{7}{2} = 7$ cm

$\therefore$ The height of the cone $= 7$ cm

$\therefore L^2 = R^2 + H^2$

$= (\frac{7}{2})^2 + (7)^2$

$= \frac{49}{4} + \frac{49}{1}$

$= \frac{245}{4}$

$$\therefore\ L = \frac{\sqrt{245}}{4} = \frac{\sqrt{245}}{2} \approx \frac{15.6}{2}\ \text{cm}$$

$$= 7.8\ \text{cm}$$

$$V = \tfrac{1}{3}\,\pi\,R^2\,H$$

$$= \tfrac{1}{3} \times \tfrac{22}{7} \times \tfrac{7}{2} \times \tfrac{7}{2} \times \tfrac{7}{1}$$

$$= \frac{539}{7 \times 2}\ \text{cu cm}$$

$$\approx 89.83\ \text{cu cm}$$

$$S_t = \pi\,R\,(R + L)$$

$$= \tfrac{22}{7} \times \tfrac{7}{2}\left(\tfrac{7}{2} + 7.8\right)$$

$$= 11 \times 11.3\ \text{sq cm}$$

$$\therefore\ S_t = 124.3\ \text{sq cm}$$

3. The total surface area of a right circular cone is 71.28 sq cm If the diameter of the base is 5.6 cm, find its volume.

The radius of the base of the cone $= \frac{5.6}{2} = 2.8$ cm

$$S_t = \pi\,R(R + L)$$

$$\therefore\ 71.28 = \tfrac{22}{7} \times 2.8(2.8 + L)$$

i.e. $\frac{22}{7} \times 2.8\,(2.8 + L) = 71.28$

$$\therefore\ 8.8\,(2.8 + L) = 71.28$$

$$\therefore\ 2.8 + L = \frac{71.28}{8.2} = 8.1$$

$$\therefore\ L = 8.1 - 2.8$$

$$= 5.3\ \text{cm}$$

$$H^2 = L^2 - R^2 \quad [\text{From } L^2 = R^2 + H^2]$$

$$= (5.3)^2 - (2.8)^2$$

$$= 28.09 - 7.84$$

$$= 20.25$$

$$\therefore\ H = \sqrt{20.25} = 4.5\ \text{cm}$$

$$V = \tfrac{1}{3}\,\pi\,R^2\,H$$

$$= \tfrac{1}{3} \times \tfrac{22}{7} \times 2.8 \times 2.8 \times 4.5$$

$$= 36.96\ \text{cu cm}$$

4. A circus tent whose lower part is right circular cylinder type and upper part is right circular cone type. The diameter of the base of the tent is 56 m and the height of the right circular cylinder is 15 m. The total height of the tent is 60 m. How many sq m of canvas is required for the tent. Find the volume of the air-space.

Solution:

Total height of the tent is 60 m and the height of the cylindrical part is 15 m. Therefore the height of the conical part is 45 m.

$L^2 = R^2 + H^2$ [Here $R = 28$, $H = 45$]

$= (28)^2 + (45)^2$

$= 784 + 2025$

$= 2809$

$\therefore L = 53$ m.

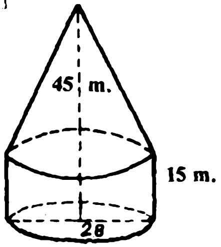


Fig. 7.15

The area of canvas required for the tent

$= 2 \pi R H + \pi R L$

$= \pi R (2 H + L)$

$= \frac{22}{7} \times 28 (2 \times 15 + 53)$

$= 88 \times 83$

$= 7304$ sq cm

$V = \pi R^2 H + \frac{1}{3} \pi R^2 h.$

['h' is the height of the conical part]

$= \pi R^2 (H + \frac{1}{3} h)$

$= \frac{22}{7} \times \frac{28}{1} \times \frac{28}{1} (15 + \frac{1}{3} \times 45)$

$= 22 \times 4 \times 28 \times 30$

$= 73820$ cu cm

Exercise 7.4

S_c = Curved surface

S_t = Total surface.

V = Volume.

R = Radius of the base.

H = Height.

L = Slant height.

Note: If the value of π is not given then substitute $\pi \approx \frac{22}{7}$

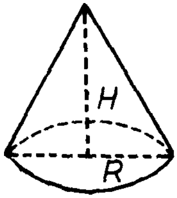


Fig. 7.16

1. From information given below in the table find by calculations other values which are asked.

Sr. No.	R	H	L	S_c	S_t	V
(*a*)	3 cm	4 cm	?	?	?	?
(*b*)	12 cm	?	13 cm	?	?	?
(*c*)	?	15 cm	17 cm	?	?	
(*d*)	4 dm	9 cm	?	?	?	?
(*e*)	21 cm	?	?	?	?	2310 cu cm
(*f*)	?	12 cm	?	?	?	616 cu cm

2. Weight of a solid right circular metal cone is 4620 gm. The density of the metal is 7.5 gm per cu cm. If the height of the right circular cone is 12 cm, find the radius of its base.
3. The density of wood is 0.8 gm per cu cm. The weight of a wooden right circular cone is 123.2 gm and its height is 3 cm. Find its curved surface area and the radius of its base.
4. The curved surface of a right circular cone is 880 sq dm. Its slant height is 20 dm. Find its volume.
5. The ratio of the radii of two right circular cones of the same height is 1:2. Find the ratio of their volumes.
6. The total surface area of a right circular cone is 770 sq cm and its slant height is four times the radius of its base. Find the diameter of the base.
7. The curved surface area of a right circular cone is 2200 sq cm. The circumference of its base is 88 cm. Find its volume.
8. The height of a right circular cone is 24 cm and the area of its base is 154 sq cm. Find the area of its curved surface, its total surface area and its volume.
9. A right circular conical tent whose base diameter is 4.8 m and whose height is 3.2 m is to be made of cloth. How many square meter of cloth is required for the tent ?
10. 176 persons can be accomodated in a right conical tent with circular base. If space available for each person is 1.26 sq m and if the height of the central pillar of the tent is 6 m, how many cubic meter of air is available for each person ?
11. The height of a right circular cylinder is equal to the diameter of its base. Find its volume.

12. A vessel has a right circular cylindrical shape. The radius of the base is 25 cm; and the height of the vessel is 40 cm. It is completely filled with milk. The milk is to be distributed to students by means of a right circular conical flask radius of whose base is 5 cm and whose height is 8 cm. How many students can be supplied with milk if each student is supplied one flask of milk ?
13. The lower part of a toy is a right circular cylinder and its upper part is right circular cone. The total height of the toy is 8 cm and that of the cylindrical part is 5 cm. Find the area of curved surface of the toy and also its volume,if its radius is 4 cm.
14. The lower part of a circus tent is a right circular cylinder and its upper part is a right circular cone. The radius of its base is 70 m. If the total height of the tent is 15 m and that of cylindrical part is 3 m find the area of the canvas required for the tent.
15. A wooden right circular cone has a base of radius 3 cm, and has height 4 cm. The upper part of the cone is cut in such a way that the conical piece will have height equal-to 2 cm and have base of radius 1.5 cm. Find the volume of the remaining portion.

Miscellaneous Exercise - 7

1. The length, breadth and height of a right rectangular cylinder of a certain metal are 44 cm , 21 cm and 12 cm respectively. By melting it, a right circular cone of the height 24 cm is formed. What is the radius of its base ?
2. The diameter of a right circular cylinder type bucket is 21 cm and its height is 40 cm. The bucket is full of sand. If the sand is poured on the ground it forms a right circular cone of height 15 cm. Find the area of the ground on which sand cone stands.
3. Find the ratio of the volumes of right circular cylinder and right circular cone whose base areas and heights are the same respectively.
4. The radius of the base of a metallic right circular cylinder is 15 cm and its height is 7 cm. The right circular cones,the diameter of whose base is 3 cm and whose height is 1.4 cm are prepared by using the total metal. How many such righ circular cones can be prepared ?
5. An ice-cream pot has a right circular cylindrical shape. The radius of its base is 12 cm and its height is 7 cm. This pot is completely filled with ice-cream; the entire ice-cream is sold in the form of right circular ice-cream cones, diameter of whose circular open portion is 4 cm and whose height is 3.5 cm. How many ice-cream cones were sold ?

6. Right circular cylinder and right circular cone have the same base of 100 sq cm area. The right circular cone is placed on the right circular cylinder. The height of the right circular cylinder is 3 cm. and total volume of the three dimensional figure is 500 cu cm Find its total height.
7. A piece of paper whose dimensions are 44 cm and 10 cm can cover completely the curved surface of a right circular cylinder, diameter of whose base is 10 cm. Find its volume.
8. From the material of a right circular cone whose base has diameter 56 cm and whose height is 15 cm a 2 mm thick wire is made. What is its length ?
9. There is a right circular cylindrical vessel, the diameter of whose base is 40 cm and whose height is 70 cm. $\frac{3}{4}$ th part of the vessel contains water. A right circular conical solid of the same base and the same height is dropped in the vessel. How much water will flow out from the vessel ?
10. If the radius of the base of a right circular cylinder is increased by 50%, then how much percentage increase will be there in its volume ?

□ □ □

ANSWERS

Chapter 1

Miscellaneous Exercise: - 1

(1) $10\sqrt{2}$ **(2)** $8\sqrt{2}$ **(3)** $4\sqrt{3}$ **(4)** $a\sqrt{2}$ **(5)** Height $= \frac{a\sqrt{3}}{2}$, Area $= \frac{\sqrt{3}}{4}a^2$ **(6)** 40 **(7)** 18 **(8)** $l(AB) = 17.66$. Area $= 54.15$ **(10)** $d = 2$, $A(\Delta PQR) = 12\sqrt{5}$ **(13)** 5. (11, 60, 61), 6. (13, 84, 85), 7. (15, 112, 113) **(15)** $\sqrt{10}$ **(16)** $l(PQ) = 13$, $l(RT) = 4.62$ **(17)** 12 **(18)** 5 **(19)** 504 **(21)** 180 **(22)** 3.5 **(23)** 14 **(24)** 96 **(25)** 192

Chapter 2

Exercise: 2.1

(1) (*a*) False. Coplanar points. (*b*) False. on the. (*c*) False. infinte.

(3) Hint: $l(PQ) < l(PR) < r$ **(7)** Hint: Perpendicular bisector of seg AB intersects L at P

Exercise: 2.2

(1) 8 **(2)** $6\sqrt{3}$ **(3)** $4\sqrt{5}$ **(4)** $2\sqrt{3}$ **(5)**(i) $10\sqrt{3}$ (ii) 5 **(6)** 3 **(7)** $5\sqrt{2}$ **(10)** 6 **(17)** 2.5

Exercise: 2.3

(3) (*a*) False. perpendicular to radius at its outer end. (*b*) True **(c)** True.

(5) (i) $10 + 10\sqrt{3}$ (ii) $25\sqrt{3}$

(10) Hint: show distance from the centre to the line AB is less than the radius.

Exercise: 2.4

(1) (i) zero (ii) one (iii) two **(2)** (i) one (ii) one (iii) two (iv) zero

(6) (i) $\frac{10\sqrt{3}}{3}$ (ii) $\frac{100\sqrt{3}}{3}$ **(10)** 30 **(13)** 50 **(15)** 9 each **(16)** 2, 3, 4 **(18)** $\frac{25\sqrt{3}}{2}$

Miscellaneous Exercise - 2

(1) $\sqrt{3}$, $2\sqrt{3}$ **(2)** 5 **(4)** 15 **(7)** $25\sqrt{3}$

Chapter 3

Exercise: 3.2

(1) (i) 100°, 40°, 40° (ii) 100, (iii) 260 **(2)** (i) 80, (ii) 140, (iii) 220

(5) (i) 50; (ii) 50, **(6)**(i) 200, (ii) 160, (iii) 100 (iv) Arc ASD

(7) (i) 200 (ii) 80, **(8)** $m\angle A = m\angle C = 90°$; $m\angle B = 72°$; $m\angle D = 108°$

(10) (i) 60, (ii) 150, (iii) 60 **(11)**(i) 142 (ii) 218 (iii) 19

(12) 105, **(13)** 80, **(14)** 50, 50

Exercise: 3.3

(1) (1) 7, 22, $\frac{77}{2}$, (2) $\frac{7}{22}$, 2, $\frac{7}{22}$, (3) $\frac{1}{2}$, 1, $\frac{11}{14}$ (4) 0.7, 1.4, 4.4.

(2) (1) $\frac{5\pi}{6}$, $\frac{25}{12}\pi$ (2) $\frac{25}{12}\pi$, $\frac{125}{48}\pi$, (3) 120, 18.48
(4) 20 π, 300 (5) 10, $\frac{125}{6}\pi$, (6) 5, 72 (3) 12 π, 48 π

(4) 50 π, 25 π, (5) 24 $\pi - 36\sqrt{3}$ i.e. 13.008 (6) 5.76 (7) 28 m.
(8) 54.2 sq.cm. (9) Rs. 11088 (10) 0.1386 sq.km. (11) 378 sq.cm. (12) 10 m.
(13) 105 m, (14) $\sqrt{\frac{2506}{11}}$ (15) 0.7854 (16) 17 m. (17) $\sqrt{3} - \frac{\pi}{2}$
(19) 0.3 (20) 35.64 Km/hr

Miscellaneous Exercise - 3

(14) 57.27 (15) 6.928 (16) 3 (17) 8.049 (18) $10\sqrt{3}$, $10\sqrt{2}$, 10

Chapter 5

Exercise: 5.1

(1) $l(PQ) = 7$, $l(QR) = 8$, $m\angle P = 70°$, $m\angle Q = 60°$,
$m\angle R = 50°$, $l(LM) = 6$, $l(MN) = 8$, $m\angle M = 90°$
(2) $m\angle A = (90 - x)$, $m\angle C = x°$, $m\angle P = (90 - x)$,
$l(MN) = 6$, $l(PN) = 8$, $l(BC) = 3$
(3) $l(QR) = 8$, $m\angle Q = 25°$, $m\angle B = 120°$, $m\angle C = 35°$
(4) $l(AC) = 18$, $l(DE) = 4$

Exercise: 5.2

(1) $\frac{3}{4}$ (2) $\triangle ABP$, $\triangle APC$, $\triangle ABC$, $\frac{5}{9}$, $\frac{5}{14}$, $\frac{9}{14}$

(3) $\triangle AQB$, $\triangle APB$, $\triangle ARB$, $\frac{3}{4}$, $\frac{1}{4}$, 4, 1. (4) $\frac{2}{5}$ (7) 2, $\frac{1}{4}$, 1, 1, 1, $\frac{2}{7}$

(8) 3:1 (9) $\frac{l(AM)}{l(CN)}$, $l(CN) = 5$ (10) $\frac{21}{2}$

Exercise: 5.3

(1) 8 (2) 15 (3) $p = \frac{24}{5}$ (5) yes. (6) 9

Exercise: 5.4

(1) 12, $\frac{24}{5}$, (2) 60, (3) 80, 120, 160, (5) 12

Exercise: 5.5

(11) $l(OM) = 2$

Exercise: 5.6

(1) $4\sqrt{5}$, $6\sqrt{5}$, 12 (2) 4 (3) 144

Exercise: 5.7

(1) Yes, Yes, No

Exercise: 5.8

(1) (i) 12, (ii) 4, (iii) 10, (iv) $\frac{3}{2}$

(2) 12, $2\sqrt{91}$ **(4)** $m = 4$

Miscellaneous Exercise - 5

(1) (i) F (ii) F (iii) T (iv) F (v) T (vi) F **(3)**(i) to (iv) $\frac{3}{4}$ (v) $\frac{9}{4}$

(4) 14, **(5)** 6, **(6)** $\frac{80}{3}$ **(11)** $\frac{16}{25}$, **(12)** 15, **(13)** 45 sq. units.

(14) 25, 35, 40 **(15)** *(a)* $l(\mathrm{ED}) = 5$, $l(\mathrm{CE}) = 14$, $l(\mathrm{EB}) = 6$ *(b)* 8, 12, 5 *(c)* 12, $\frac{15}{2}$, 5

Chapter 6

Exercise: 6.1

(1) $\frac{3}{5}$, **(2)** $\frac{9}{41}, \frac{9}{40}$ **(3)** $\frac{5}{13}, \frac{5}{12}$ **(4)** $\frac{1}{3}, \frac{2\sqrt{2}}{3}$

(5) $\frac{2\sqrt{6}}{7}, \frac{5}{2\sqrt{6}}$ **(6)** $\frac{1}{2}, \frac{\sqrt{3}}{2}$ **(7)** $\frac{\sqrt{7}}{4}$ $\frac{\sqrt{7}}{3}$ **(8)** $\frac{20}{21}, \frac{21}{29}, \frac{20}{29}$

(9) $\frac{31}{25}, \frac{168}{625}$ **(10)** $\frac{40}{41}, \frac{9}{41}$ **(11)** $\frac{11}{61}, \frac{60}{11}$ **(12)** 5, $5\sqrt{3}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$

(13) $\frac{2}{\sqrt{13}}$ **(14)** 15, $\frac{3}{5}, \frac{3}{4}$ **(15)** 15, 20, $\frac{4}{5}, \frac{3}{4}$

Exercise: 6.2

(1) $\frac{1}{8}$ **(2)** $\frac{4}{3}$ **(3)** $\frac{3}{4}$ **(4)** $1\frac{3}{4}$

(5) $\frac{2\sqrt{3}}{1+\sqrt{3}}$ **(6)** $\frac{2\sqrt{3}}{1-\sqrt{3}}$ **(7)** $\frac{\sqrt{3}}{2}$ **(8)** $\frac{3}{2}$

(9) 15 **(10)** 2 **(11)** $\frac{1}{2}$ **(12)** 5

(13) $\frac{1}{2}$ **(14)** 60° **(15)** $\frac{2}{3}$ **(16)** $4\frac{1}{3}$

(17) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **(18)** 0 **(19)** $\frac{\sqrt{3}}{2}$ **(20)** $-3\frac{3}{5}$

(21) 4 **(22)** 1 **(30)** 200, $100\sqrt{3}$ **(31)** $\frac{50\sqrt{3}}{3}, \frac{100\sqrt{3}}{3}$

(32) 66 **(33)** 165.89

Exercise: 6.3

(1) $20\sqrt{3}$ m. **(2)** 20.8 m **(3)** 25.38 m

(4) 539.77 m **(5)** 24·7 (approx.) **(6)** 40 m, $20\sqrt{3}$ m

(7) 36.50 m **(8)** 10.95 m **(9)** 630 m (approx.)

(10) 100 m

Miscellaneous Exercise: 6

(1) $\frac{l(MC)}{l(BC)}$, $\frac{l(MB)}{l(MC)}$ (2) 9.06, 4.23, 14.50, 18.73 (3) 8.39, 13.42 (4) 35.66

(5) (A) (1)BAD (2) BCD (3) BAD (B) < (6) $x = 3.42$, $r = 9.40$, $y = 25.80$

$z^2 = 754$ (8) $\frac{15}{17}$, $\frac{8}{15}$ (15) $\frac{17}{4}$ (17) $2\frac{6}{25}$ (18) 1

(19) $\frac{1}{2}$ (21) 60° (22) 30 m, $10\sqrt{3}$ m (23) 2.732 km.

Chapter 7

Exercise 7.1

(1) 50 sq.cm. (2) 10 m (3) 5.2 cm (4) 600 cu.cm. (5) 10 cm (6) $90\sqrt{3}$ cu.cm.
(7) $3000\sqrt{3}$ cu.cm, 1200 sq. cm.

Exercise 7.2

(1) (*a*) 440 sq.cm., 1672 sq.cm (*b*) 1980 sq.cm. 4752 sq.cm (*c*) 70400 sq.cm., 90112 sq. cm. (*d*) 1320 sq. dm, 1763.52 sq. dm (*e*) 24.64 sq. m. 36.96 sq. m.

(2) 506.88 sq.cm. (3) 174.24 sq.cm., 201.96 sq. cm.
(4) 7 (5) Rs. 105 - 60
(6) 75 m (7) Rs. 72 - 60
(8) 1.225 m (9) Rs. 13.20
(10) Rs. 35200 (11) 1100 sq.m.
(12) 1200 (13) 40
(14) 12.69 m (15) 42372 sq. cm.

Exercise: 7.3

(1) (*a*) 231 cu.cm (*b*) 17.6 cu.cm (*c*) 2.2 cu.m. (*d*) 45 cm (*e*) 14 cm.
(2) 13.5 (3) 331.5 cu.cm more water in square base vessel. (4) 38500 litre. (5) 38016
(6) 462 cu.cm. (7) 9240 cu.m. (8) Rs. 492.80 (9) 1131.43 cu.cm
(10) 20 m and .5 sec. (11) 10 m (12) 25 m. (13) 44 bags (14) 45 (15) 144
(16) 13 hour 7.5 minutes. (17) $179\frac{2}{3}$ cu.cm, $51\frac{1}{3}$ sq.cm (18) 5 cm, 7 cm

Exercise: 7.4

(2) 7 cm (3) 167.55 sq.cm. (4) 2932.7 (5) 1:4 (6) 14 cm (7) 9856 sq.cm
(8) 550 sq.cm., 704 sq.cm., 1232 cu.cm. (9) 30.144 sq.m. (10) 2.52 cu.m. (11) $\frac{2\pi R^2}{3}$
(12) 375 (13) 188.57 sq. cm., 301.71 cu.cm. (14) 4730 sq.m. (15) 33 cu.cm.

Miscellaneous Exercise - 7

(1) 21 cm (2) 2772 sq.cm.
(3) 1:3 (4) 1500
(5) 216 (6) 9
(7) 1100 cu.cm. (8) 3920 m
(9) 7333.3 cu.cm. (10) 125%

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